MECHANISM

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MECHANISM

BY

ROBERT McARDLE KEOWN, B. S.
ASSISTANT PROFESSOR OF MACHINE DESIGN
UNIVERSITY OF WISCONSIN

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PREFACE

The writer's aim has been to cover the subject of mechanism as briefly, simply and clearly as possible. The text is designed for a half year's work of one lecture, one recitation and four hours' drafting work per week.

No especial claim to originality of subject matter can be made, nor has the writer found need for a special effort in this direction. The arrangement and method of treatment are new, and these the author bases upon satisfactory results obtained in the work of his classes in the College of Engineering of the University of Wisconsin.

After a brief discussion of motions and velocities linkages are taken up, as they are comparatively easy for the student to understand, and simple problems can be given out while the subject is yet new to h m.

Cams are taken up in detail, as they form a part of the subject in which the student needs considerable practice in order to work out original problems, and practically all cam problems are original.

The involute system of gearing is taken up before the cycloidal system, because it seems to the writer easier for the student to grasp. Having once become familiar with the involute system, the student can more readily understand the cycloidal. Furthermore, the involute system is in more general use at the present time.

Problems are given at the end of each chapter. Some of these are designed especially for drafting-room work, and the necessary instructions as to scale, position of drawing on the sheet, method of procedure and time, etc., are given. These few problems are not intended to exhaust the subject, but rather to serve as a guide for instructors in giving the drafting-room work in connection with the recitations.

Quotations have been made from the various authorities, and credit given where the quotations appear. The writer wishes to express his thanks to the several authors from whom he has made quotations, the manufacturing companies who have furnished cuts and to all others who have aided in the preparation of the manuscript.

MADISON, WIS.,
August, 1912.

R. McA. KEOWN.

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MECHANISM

CHAPTER I

MOTIONS AND VELOCITIES

The science of Mechanism treats of the design and construction of machinery.

"A *machine* is a combination of resistant bodies so arranged that by their means the mechanical forces of nature can be compelled to produce some effect or work accompanied by certain determinate motions," *Realeaux*. In general it may be said that a machine is an assemblage of moving parts interposed between the source of power and the work for the purpose of adapting one to the other.

1. Design of a Machine.—In the design of a machine there are three distinct parts to be considered:

First.—The general outline is sketched without any regard to the detailed proportions of the individual parts, and from this sketch or skeleton, by means of pure geometry alone, the displacement, velocity, and acceleration of each of the moving parts can usually be accurately determined.

Second.—The force acting on each part has to be determined, and each part given its proper form and dimensions to withstand these forces.

Third.—Having designed the machine, the dynamical effects of moving parts can be accurately determined.

The first part belongs to the subject known as the Kinematics of Machines, the second to the Design of Machine Parts, and the third to the Dynamics of Machines.

This course will treat only of the first part dealing with the motions of the machine parts, and the manner of supporting and guiding them without regard to their strength. This is sometimes called Pure Mechanism or the Geometry of Machinery.

Since Mechanism is a study of relative motion it will be well to discuss the different kinds of motion.

2. Motion.—Motion is a change of position. The change of

position can be noted only with respect to the position of some other body which is at rest, or assumed to be at rest, or with respect to some body, the motion of which is known or assumed to be known. That is, the motion is purely a relative one.

Two bodies may be at rest relatively to each other but in motion relatively to a third body, as for example, two car wheels fastened to the same axle have no motions with respect to each other, but may be in motion relatively to the truck or track.

In problems dealing with machinery the motions of the various parts are usually taken with reference to the frame of the machine. This is not always the case however, as sometimes it is easier to compare the motion of one part of a machine directly with that of some other moving part, as for example the number of revolutions that the cylinder of a printing press makes to each revolution of the knife for cutting off the sheets.

3. Forms of Motion.—In order to be of any use in machine construction, the motions must be completely controlled or constrained. Most of the motions used are, or can be reduced to one of three forms, *plane*, *helical* and *spherical*.

4. Plane Motion.—Plane motion is the most common as well as being the most simple. In order to have plane motion all points in a plane section of the body must remain in that plane and all points outside that section will move in parallel planes.

In Fig. 1, if the lower face *abcd* of a cube is kept in contact with

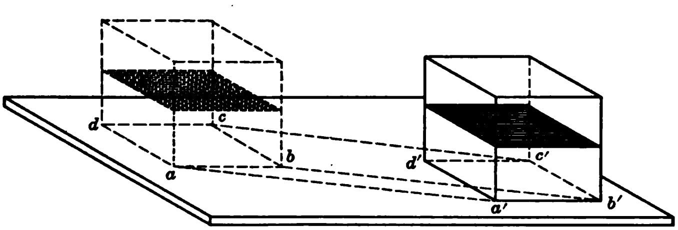


FIG. 1.

a flat table top, all points in the lower face will move in a plane, and all points similarly located in any other plane section will move in parallel planes, no matter what path the cube describes in moving from one position to another.

Thus if we know the motions of two points in a body that has plane motion, the motions of other points in the body are also known.

Plane motion is always found as *rotation* or *translation*, or a motion that can be reduced to a combination of rotation and translation.

5. Rotation.—In rotation all points in the body move in circles, that is, they remain at a fixed distance from a right line called the axis of rotation. For example, shafts, pulleys, fly-wheels, etc., have a motion of rotation.

6. Translation.—Translation may be divided into two classes, *Rectilinear* Translation and *Curvilinear* Translation. A body has rectilinear translation when all points in it move in straight lines, as the carriage of a lathe, piston of an engine, or spindle of a drill press.

The crank pin of a locomotive driving wheel is an example of curvilinear translation. It moves in a circle around the center of the wheel and at the same time moves along the track.

7. Helical Motion.—The path traced by a point moving at a fixed distance from an axis and with a uniform motion along the axis is a helix, and a point moving in such a path is said to have helical motion.

Perhaps the most common example of helical motion is that of a screw being turned into a nut. All points in the screw, except those in the axis, have helical motion. Both limits of helical motion are plane motion, that is, if the pitch of the screw is zero the resulting motion is rotation, while if the pitch is infinity, the motion will be that of translation.

8. Spherical Motion.—Spherical motion may be defined as the motion of a body moving so that every point in it remains at a constant distance from a center of motion, but does not remain in a plane.

9. Path.—A point in changing from one position to another traces a line called its path. The path may be of any form. The path traced by any point on a pulley revolving on its shaft is a circle, but a path need not be continuous, as for example the path traced by a rifle bullet.

In general, the motion of a body is determined by the paths of three of its points not in a right line. If the motion is in a plane, two points are sufficient, and if rectilinear, one point determines the motion.

10. Velocity.—In addition to knowing the path and direction of a moving body, there is another element necessary to completely determine its position, and that is its *velocity*.

Heretofore we have not considered the *time* necessary for a body to complete a certain motion.

Velocity is measured by the relation between the space passed over and the *time* occupied in traversing that distance. It is expressed numerically by the number of units of distance passed over in one unit of time, as miles per hour, feet per minute, inches per second, etc. It is the *rate of motion* of a point in space.

11. Linear Velocity (V^-).—When a motion is referred to a point in the path of a body, its velocity is expressed in linear measure and is called its *linear velocity.*

Velocity is uniform when equal spaces are passed over in equal units of time; that is, the *distance varies as the time.*

Let V = velocity; S = total distance passed over; T = time occupied.

Then if we know the distance passed over by a body and the time it occupied in passing over that distance, the velocity, $V = \frac{S}{T}$. Thus if a body with a uniform velocity passes over a distance of 50 ft. and occupies 5 minutes in doing so, it has a velocity of $\frac{50}{5} = 10$ ft. per minute.

If the velocity and the time occupied are known, the space passed over, $S = V \times T$, while if the space and velocity are known, the time $T = \frac{S}{T}$.

12. Variable Linear Velocity.—A body which has a motion that is accelerated or retarded is said to have a variable linear velocity. Such a velocity is not constant, but when calculating the velocity at any point in its path, its velocity is assumed to be constant for that instant. Thus a train starting from rest and gradually increasing its speed until it attains a velocity of 60 miles per hour, has a variable velocity. After the train had gone a distance of one mile from its starting-point, it might have a velocity of 15 miles per hour, meaning that if its velocity remained constant, it would go 15 miles in the next hour.

13. Angular Velocity (V°).—The angular velocity of a point is the number of units of angular space which would be swept over in a unit of time by a line joining the given point, with a point outside its path, about which the angular velocity is desired.

The angular space is measured in circular measure, or it is the ratio of the *arc* to the *radius.*

The unit angle or the *radian* is one subtended by an arc equal in length to the radius, r.

Hence in one circumference there are $\frac{2\pi r}{r}=2\pi=6.2832$ radians, or a radian $=\frac{360°}{6.2832}=57.29°$.

In engineering practice, angular velocity is usually expressed in revolutions per unit of time.

Thus if a point S in Fig. 2 makes n revolutions per unit of

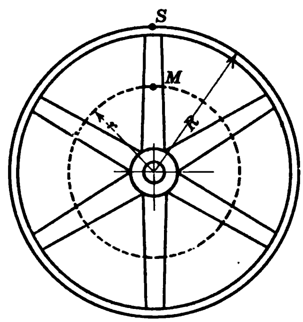


FIG. 2.

time, its angular, $V°=2\pi n$ radians per unit of time. This is the angle that a line joining S with the center of the pulley will pass through per unit of time.

14. All points in the pulley of Fig. 2 have the same angular velocity, since they all pass through the same angle in a unit of time.

Also all points in the pulley having the same radii will have the same linear velocity, but the greater the radius the larger the linear velocity. Thus if the point M on the pulley has a radius r, the $V^{-}M=2\pi rn$ while the $V^{-}S=2\pi Rn$. $V^{-}S:V^{-}M=2\pi Rn:2\pi rn$

$$\text{or } \frac{V^{-}S}{V^{-}M}=\frac{2\pi Rn}{2\pi rn}=\frac{R}{r}$$

Hence we may state the following rule:

The linear velocities of all points in the same body are directly proportional to their distances from the center of rotation of the body.

15. Relation between Linear Velocity and Angular Velocity.— The angular velocity of a point in a rotating body is $2\pi n$ radians

per unit of time and the linear velocity of a point in the same body is $2\pi rn$ or the angular velocity is the same as the linear velocity of a point in the body having a *unit radius.*

We may then say that the angular velocity $= \frac{\text{linear velocity}}{\text{radius}}$ or $V^\circ = \frac{V^-}{r}$. This gives the angular velocity in radians and to reduce it to revolutions, divide by 2π.

This relation is often used and should be remembered. *The angular velocities of two points in different bodies having the same linear velocities, but different radial distances from their centers of rotation, are inversely proportional to their radii.*

Let P and S, Fig. 3, be the two points in A and B respectively

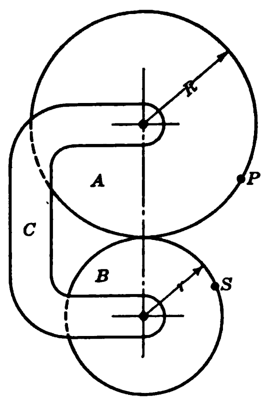


FIG. 3.

in which R is the radius of P from its center of rotation and r the radius of S from its center of rotation.

$$\text{Let } V^-P = V^-S.$$

$$V^\circ A = \frac{V^-P}{R};\ V^\circ B = \frac{V^-S}{r}$$

$$\text{Then } V^\circ A : V^\circ B = \frac{V^-P}{R} : \frac{V^-S}{r}$$

$$\text{or } \frac{V^\circ A}{V^\circ B} = \frac{\frac{V^-P}{R}}{\frac{V^-S}{r}}$$

$$\text{But } V^-P = V^-S \ \therefore\ \frac{V^\circ A}{V^\circ B} = \frac{r}{R}$$

PROBLEMS

1. An engine piston makes 400 single strokes per minute; the flywheel is on the crank shaft. What is the linear velocity of the crank pin if the length of the crank is 15 in.?

2. A pulley makes 400 r.p.m. and a point on its rim has a linear velocity of 4000 ft. per minute. How far is the point from the center of rotation?

3. A flywheel 28 ft. diameter makes 30 r.p.m. On the flywheel shaft is a crank of 15 in. radius which transmits motion to a crosshead by means of a connecting rod 6 ft. long. Through how many feet does the crosshead travel per minute?

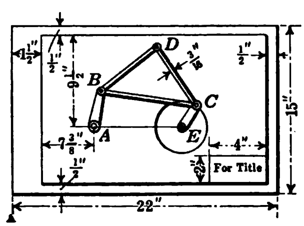
4. Two wheels A and B are in contact and roll together without slipping. The distance between their centers is 30 in. A makes 80 r.p.m. and B 300 r.p.m. What are their diameters?

5. In two wheels A and B which roll together without slipping, their diameters are 16 in. and 12 in. respectively, and B makes 125 r.p.m. What are the r.p.m. of A?

6. Velocity ratio to be transmitted is as 3:4. Diameter of driven 20 in. Find the diameter of the driver and the distance between the center of two wheels.

7. Plot the path of D for one complete revolution of the crank EC.

Data: $AE = 6''$; $BC = 6\frac{1}{4}''$; $BD = CD = 5\frac{1}{2}''$; $AB = 2\frac{3}{16}''$; $EC = 1\frac{1}{8}''$.
Diameter of hubs A and $E = \frac{1}{2}''$
Diameter of hubs B, C and $D = \frac{3}{8}''$
Diameter of pins A and $E = \frac{1}{4}''$
Diameter of pins B, C and $D = \frac{3}{16}''$



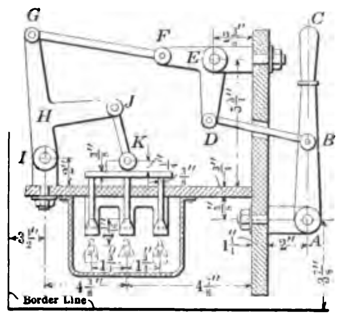
Note.—Make an ink drawing, full size, placing the point A as indicated. Divide the circle, the radius of which is EC, into 12 equal parts, numbering them consecutively, and also number the corresponding points on the path of D.

The linkage is to be drawn in black and other lines in red. Put no dimensions on the drawing.

The statement of the problem and data are to be placed in the upper right-hand corner of the sheet. Time, 4 hours.

8. In the hand-operated oil switch shown, through what angle must the handle AC move in order to completely open or close the switch?

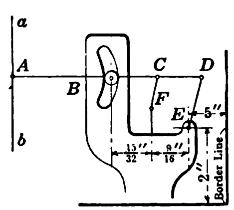
Data:	Links	AB	BC	BD	DE	EF	FG	GH	HI	HJ	JK
	Length	$3\frac{1}{4}''$	$8''$	$6\frac{5}{8}''$	$3\frac{1}{2}''$	$3''$	$4\frac{3}{4}''$	$4''$	$2''$	$4\frac{1}{4}''$	$2\frac{1}{2}''$
	Hubs	A	B	D	E	F	G	I	J	K	
	Diameter	$\frac{7}{8}''$	$\frac{5}{8}''$	$\frac{5}{8}''$	$\frac{7}{8}''$	$\frac{5}{8}''$	$\frac{5}{8}''$	$\frac{7}{8}''$	$\frac{5}{8}''$	$\frac{5}{8}''$	
	Pins, Diameter	$\frac{1}{2}''$	$\frac{5}{16}''$	$\frac{5}{16}''$	$\frac{1}{2}''$	$\frac{5}{16}''$	$\frac{5}{16}''$	$\frac{1}{2}''$	$\frac{5}{16}''$	$\frac{5}{16}''$	



Note.—Make a full size ink drawing and leave off all dimensions. Draw the links in one extreme position, and represent the other extreme by center lines only. Time, 6 hours.

9. In the Tabor engine indicator plot the path of B for a motion of $3\frac{1}{8}$ in. for A along the straight line ab.

Data: Diameter of roll at $B = \frac{3}{16}''$; $AD = 3\frac{1}{8}''$; $CD = \frac{5}{8}''$; $BC = \frac{5}{8}''$; $CF = 1''$; $DE = 1\frac{1}{4}''$.



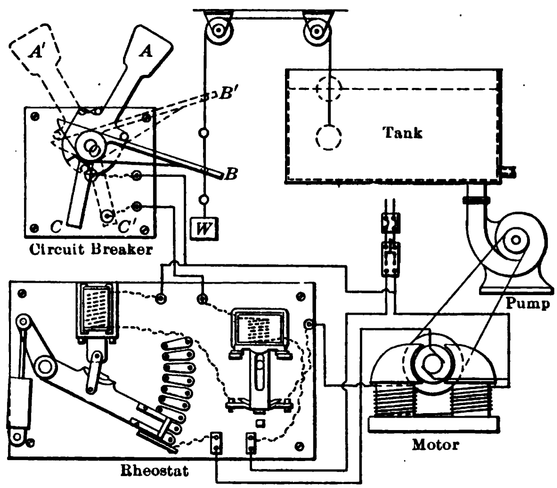
Note.—Make a pencil skeleton drawing, scale 4 in. = 1 in. Take points on line ab $\frac{1}{8}$ in. apart and find the corresponding positions of B and F. Time, 4 hours.

10. Design the circuit-breaker so that it will make or break contact for a difference of $13\frac{1}{2}$ in. in the water level in the tank. Locate the extreme positions of both balls and of all the levers. Marble slab for circuit-breaker 12 in. square.

Note.—Make an ink drawing half size, and design the different parts without regard to strength but so that they will look of the proper proportions. The tank need not be drawn to scale, although the difference

in water level must be drawn to the same scale as the switch mechanism. Do not show the rheostat, motor or pump.

The statement of the problem to be placed in the upper right-hand corner of the sheet and underneath the drawing, AUTOMATIC FLOAT SWITCH. Time, 6 hours.



CHAPTER II

INSTANTANEOUS CENTERS, KINEMATIC CHAINS.

16. Mechanisms.—A mechanism or train of mechanism is the term applied to a portion of a machine where two or more parts are combined so that the motion of the first compels the motion of the others according to a law depending on the nature of the combination. The two parts connected together are known as an *elementary combination,* so that a train of mechanism consists of a series of elementary combinations.

If a part is considered separately from the others it is at liberty to move in the two opposite directions and with any velocity, as the crosshead of an engine that is not connected to the connecting rod.

Wheels, shafts and rotating parts generally are so connected with the frame of the machine that any given point is compelled when in motion, to describe a circle around the axis, and in a plane perpendicular to it. Sliding parts are compelled by fixed guides to describe straight lines, other parts to move so that points in them describe more complicated paths and so on.

These parts are connected in successive order in various ways so that when the first part in the series is moved, it compels the second to move, which again gives motion to the third, etc. The various laws of motion of the different parts of a train are affected by the mode of connection.

17. Modes of Connection.—Connection between the different parts of a machine may be made in any of the following ways:

1. By direct contact
   - *a.* Turning pairs—connected links.
   - *b.* Slide connector—crosshead.
   - *c.* Cams without rollers.
   - *d.* Friction contact—friction cylinders.
   - *e.* Rolling and sliding contact—toothed gears.
2. By intermediate connectors
   - *a.* Cams with rollers.
   - *b.* Rigid links.
   - *c.* Flexible connectors—ropes, belts and chains.

3. Without material connectors
   - *a.* Electricity and magnetism.
   - *b.* Gravity.
   - *c.* Centrifugal force—governor.

The first two methods are perhaps the most important in this subject, and will be discussed later on.

18. Instantaneous Motion.—When a body changes its position its motion at any instant may be said to be its *instantaneous motion.*

As an illustration take the car wheel A, moving along the track B, Fig. 4. For an instant there is contact between the wheel and

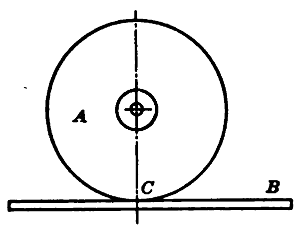


Fig. 4.

the track along a line at C perpendicular to the paper and parallel to the axis of the wheel. The wheel then rotates about C which becomes an element of both the wheel and the track for an instant, and this motion of the wheel is its instantaneous motion. The line through C and perpendicular to the paper is its *instantaneous axis.*

If instead of considering the whole wheel, we take a section made by passing a plane through it perpendicular to the axis, it will cut the line through C in a point which is called the *instantaneous center* or *centro.*

19. The relative motion between A and B is the same whether we consider the track stationary and the wheel revolving about it, or whether the wheel is considered stationary and the track revolving.

20. Centro.—A centro is a point common to two bodies having the same linear velocity in each, or it is a point in one body about which the other tends to rotate.

As an illustration of the first part of the definition, the tangent point of the two wheels A and B, Fig. 3, is an example. It is a point common to the bodies A and B and having the same linear velocity in each. The second part of the definition is illustrated

by the points in the frame *C* of Fig. 3, about which *A* and *B* rotate.

21. Location of the Centro in a Single Body.—In any rigid body moving about an axis, the direction of motion of any point is perpendicular to a line joining the point with the axis; conversely, the axis of rotation will intersect a line drawn perpendicular to the motion of any point in the body, and lying in the plane of the motion of the point.

An illustration of a special case of this statement is the motion of a point on the rim of a fly wheel. Its instantaneous direction of motion is tangent to the rim, or perpendicular to a line joining the point with the axis of rotation.

A general illustration is as follows: Let the points *M* and *N*, Fig. 5, have the directions of motion shown by the arrows, both

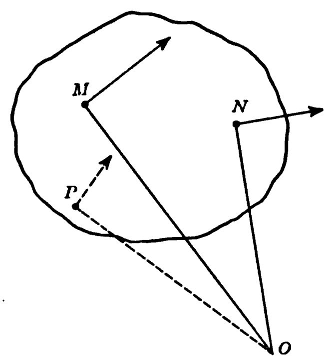


FIG. 5.

motions lying in the same plane. Lines drawn through *M* and *N* and perpendicular to their respective directions of motion, will, if they intersect, locate the centro, or instantaneous center of the two points, and of the body as a whole. In Fig. 5 these lines intersect at *O*. To determine the direction of motion of any other point as *P*, draw a line through *P*, perpendicular to the line *PO*.

In some cases the centro cannot be located, as when the lines drawn perpendicular to the directions of motion coincide or when they are parallel. In the latter case the centro is at an infinite distance away and the body has a motion of translation.

22. Location of Centros in Three Bodies Moving Relatively to Each Other.—Any three bodies moving relatively to each other

have but three centros, and *these centros lie on the same straight line.*

In Fig. 6, let *A*, *B* and *C* be the three bodies that move relatively to each other. Assume that the body *C* is held stationary, and that *A* and *B* are fastened by pin joints to *C* at *ac* and *bc* respectively.

The number and names of the centros are found by taking the bodies in combinations of two using each one with each of the

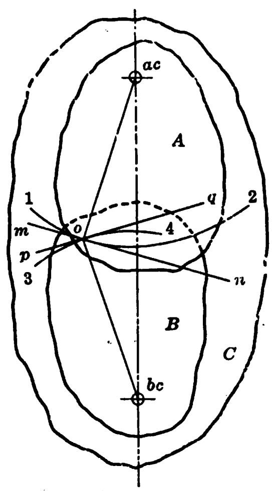


FIG. 6.

others, as *AB*, *AC* and *BC*. The bodies are usually represented by capital letters and the centros by lower case letters, so that the centros will be *ab*, *ac* and *bc*. It makes no difference in which order the letters are taken as *ab* or *ba* for the relative motion of *A* about *B* is the same as the relative motion of *B* about *A* (Art. 19).

The only motion that *A* can have is that of rotation about *ac*, while the only motion of *B* is that of rotation about *bc*. This locates two of the centros, and the third one *ab* is common to the bodies *A* and *B* and has the same linear velocity in each.

Assume that *ab* lies at the point *o*. When *o* is considered as a point in *A*, it has a radius from its center of rotation *o–ac*, and moves in the arc 1–2. Its direction of motion is perpendicular to its radius, or along the line *m–n*.

When *o* is considered as a point in *B*, it has a radius *o–bc* and

moves in the arc 3–4. Its direction of motion is perpendicular to its radius, or along the line *p–q*.

Thus we have the point *o* moving in two different directions at the same time, which is impossible. The lines *m–n* and *p–q* must be parallel or they must coincide. They cannot be parallel since they must both pass through the same point, hence they must coincide, and the only time that they will coincide is when the radii *o–ac* and *o–bc* lie in the same straight line.

23. Kinematic Chain.—A kinematic chain is a combination of rigid bodies or links so connected that the motion of each is completely controlled by and depends upon the motions and positions of each of the others. Figs. 7, 8 and 9 are examples.

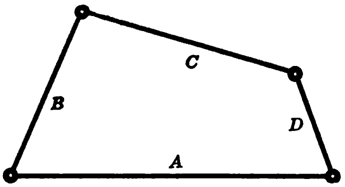


Fig. 7.

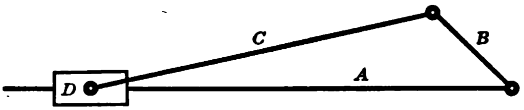


Fig. 8.

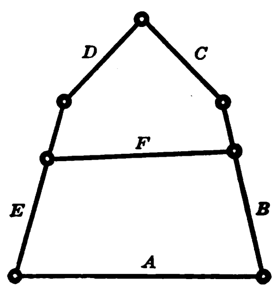


Fig. 9.

By holding one of the links stationary, any motion given to one will cause a certain definite motion in each of the others. The links can be of any shape whatever as long as their shape does not cause them to interfere with the motions of any of the other links.

Fig. 7 is a simple four-linked kinematic chain consisting of four turning pairs, while Fig. 8 is a simple chain of three turning and one sliding pair. Fig. 9 is a *compound* chain. A compound kinematic chain is one in which one or more of the links has more than two joints. In Fig. 9 the links B and E have three joints or are each connected to three other links.

24. Location and Number of Centros in a Kinematic Chain.—To find the centros in a kinematic chain write the names of the links in a row, and underneath each, its combinations with each of the others. The number of centros in any kinematic chain $=\frac{N(N-1)}{2}$ where N is the number of links in the chain.

In the four-link chain of Fig. 10, the number of centros will be

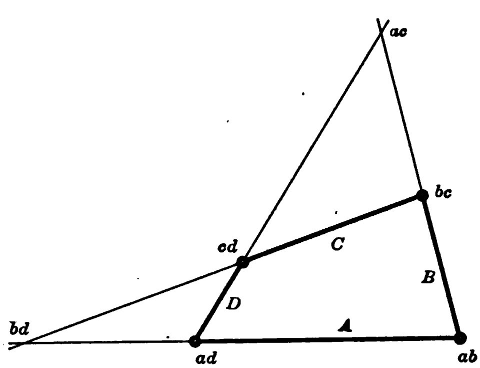


FIG. 10.

$\frac{4(4-1)}{2}=6$ centros. Their names can be found as stated above.

A	*B*	*C*	*D*
ab	*bc*	*cd*	
ac	*bd*		
ad			

Four of these centros, *ab*, *bc*, *cd* and *ad*, can be found at the joints of the links, leaving *ac* and *bd* to be located. To locate the position of *ac* we know that the three centros of any three bodies moving relatively to each other lie on the same straight line (Art. 22), and also that to get *ac* the links A and C must be used since the centro *ac* is common to links A and C. Take A and C with one of the other links, say B. Then in the links A, B, C, the centros will be *ab*, *ac*, *bc*. We have already located *ab* and *bc*,

and since *ac* also lies on the same straight line with them, we can draw a line of indefinite length through *ab* and *bc*.

Now take with the links *A* and *C*, with a link not used before, as *D*. The centros of *A*, *C*, *D*, will be *ac*, *ad*, *cd*. We already know the positions of *ad* and *cd*, and *ac* will lie on the same straight line with them. Draw a line of indefinite length through *ad* and *cd*. Now since *ac* lies on this line, and also on the line drawn through *ab* and *bc*, it must lie at the intersection of these lines.

The centro *bd* is found in a similar manner.

When there is sliding contact between two links of a kinematic chain as between *A* and *B*, Fig. 11, the centro will be at an infinite

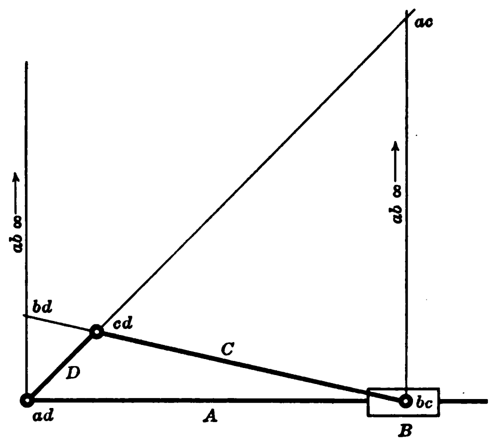


FIG. 11.

distance away, since it is the point in *A* about which *B* tends to rotate, and as *B* has a motion of translation, the point about which *B* tends to rotate is an infinite distance away, along a line perpendicular to the direction of motion (Art. 21).

The centros *ad*, *cd* and *bc* being at the joints of the links are readily found, and *bd* is at the intersection of the lines *bc–cd* and *ad–ab*.

The location of the centros in a compound chain can be found in a similar way. First locate all of those at the joints of the links and at infinity, and then taking any three of the links in which two of the centros are known, find the third one.

PROBLEMS

11. Prove that in three bodies moving relatively to each other, their three centros lie on the same straight line.

12. Assume the positions of the links in a simple crossed-link chain and locate all of the centros.

13. Locate all of the centros in Fig. 29 for the position of the links shown.

14. Locate all of the centros in Fig. 9 for the position of the links shown.

CHAPTER III

SOLUTION OF RELATIVE LINEAR VELOCITIES BY CENTRO METHOD

25. Relative Linear Velocity.—In comparing the linear velocity of any point in one link with the linear velocity of a point in any other link, the comparison must be made through a point common to both of them.

In Fig. 12 let A and B be two bodies, held in contact with each

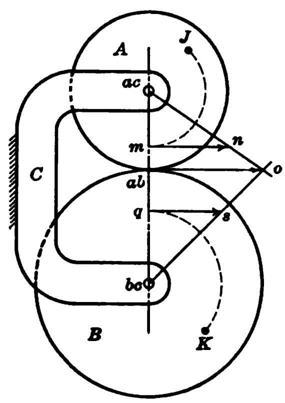


FIG. 12.

other by the frame C, which consider stationary. The centros of A, B and C are ab, ac and bc. If C is stationary the centros ac and bc will be stationary since they are the points in C about which A and B respectively tend to rotate; and ab is a point common to A and B having the same linear velocity in each.

Let J be a point on A whose radius and linear velocity are known, and let K be a point on B, whose radius is known and whose velocity it is desired to find. The point J has a radius J–ac and revolving J about its center will not affect its velocity as long as its radius is not changed. Revolve J around to the line of centers and lay off a line m–n to any convenient scale to represent its linear velocity.

The velocities of all points in the same link or body are directly proportional to their distances from the center of rotation (Art. 14), so that the linear velocity of *ab* having a radius *ac–ab* is directly proportional to the linear velocity of *J*, having the radius *J–ac*. The velocity of *ab* can now be found by similar triangles and is represented by the line *ab–o*.

Since the point *ab* is common to the bodies *A* and *B*, its linear velocity will be the same in each, although its radius, when considered as a point in *B* is not the same as it was when considered as a point in *A*. The linear velocities of *K* and *ab* are directly proportional to their radii.

Complete the triangle *bc–ab–o* by drawing the line *bc–o*, and revolve *K* around its center to the line of centers at *q*. Draw through *q* the line *q–s*, parallel to *ab–o*. Then from similar triangles, if *ab* is the linear velocity of *ab*, *q–s* to the same scale will be the linear velocity of *K*, or if *m–n* is the linear velocity of *J* then *q–s* is the linear velocity of *K* to the same scale.

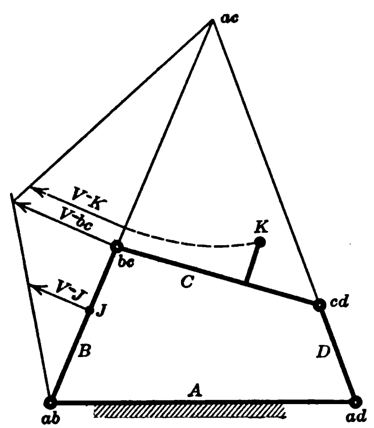


FIG. 13.

26. Bear in mind that from the linear velocity of the given point, the linear velocity of the common point must first be found and from that find the linear velocity of the required point.

27. Open Four Linked Mechanisms. *Velocities of Points in Adjacent Links.*—Let *A*, *B*, *C*, *D*, Fig. 13, be an open four-linked mechanism in which it is required to find the $V^{-}K$, a point in the link *C*, knowing the $V^{-}J$, a point in the link *B*, for the positions of the links shown and with the link *A* stationary.

It is only necessary to consider the links containing the points and the stationary link. These are *A*, *B* and *C*. The centros are *ab*, *ac* and *bc*. Since the link *A* is stationary, the centros *ab* and *ac* will be stationary and *bc* will be the common point, or the point having the same linear velocity in *B* and *C*.

The stationary centro *ab* is the point in *A* about which *B* rotates, and since *J* is a point in *B*, it also rotates about the centro *ab*.

Lay off from *J* a length of line to any convenient scale to represent the V^-J. (This line should be laid off perpendicular to the radius of *J*.)

From the V^-J find the V^-bc by similar triangles. The linear velocities of points in the same link are directly proportional to their instantaneous radii as well as their actual radii.

Revolve *K* about *ac* into the line of centers and after completing the triangle of which the V^-bc is one side, lay off the V^-K.

28. *Velocities of Points in Opposite Links.*—Given the V^-J Fig. 14 to find the V^-K with the link *A* stationary.

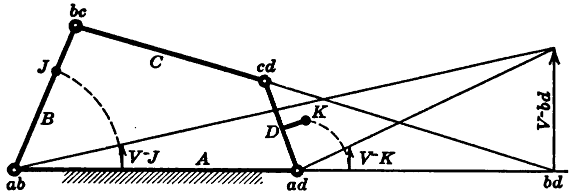


FIG. 14.

The links that must be considered are the two links containing the points and the stationary link, which in this case are *A*, *B* and *D*, having centros *ab*, *ad* and *bd*. Since the link *A* is stationary, the centros of *A* will be stationary, viz., *ab* and *ad*, and *bd* is the common point, or the point common to *B* and *D*, and having the same linear velocity in each.

Revolve *J* around into the line of centers and lay off a line to any convenient scale to represent its velocity, then by means of similar triangles find the linear velocity of the common point *bd*. From this, find the linear velocity of *K*, after revolving it around into the line of centers.

29. Crossed Link Mechanisms. *Velocities of Points in Adjacent Links.*—Let it be required to find the V^-K, Fig. 15, having given the V^-J with the link *B* stationary.

The links to be taken into consideration are A, B and C, having centros ab, ac and bc, of which ab and bc are stationary.

Lay off a line to represent the $V^{-}J$, and by means of similar triangles find the $V^{-}ac$, the common point. Then revolve K

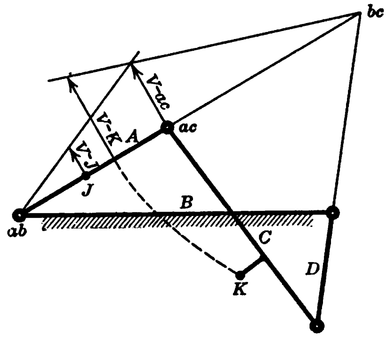


Fig. 15.

about bc into the line of centers and by means of another set of similar triangles find the $V^{-}K$ as shown.

30. *Velocities of Points in Non-adjacent Links.*—Let A, B, C and D, Fig. 16, be a crossed link mechanism in which it is desired to find the $V^{-}K$, a point in the link D, having given the $V^{-}J$ a point in the link B, with the link A stationary.

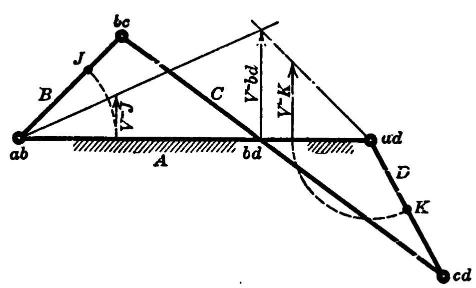


Fig. 16.

The links that need to be considered are A, B and D, having centros ab, ad, and bd, in which ab and ad are stationary and bd is the common point. Notice that bd is located at the intersection of the links A and C, but of course there is no joint at that point, for if there were, the mechanism would be as one rigid link, as there could be no motion of any of the links.

The link B rotates about the centro ab, and as J is a point in the link B, it will rotate about ab.

Revolve J into the line of centers and lay off a line to represent its linear velocity. From the $V^{-}J$, find the $V^{-}bd$ by similar triangles. K rotates about the centro ad as ad is the point in A about which D rotates. Revolve K around ad into the line of centers and find its linear velocity by similar triangles.

31. Slider Crank Mechanism. *Velocities of Points in Adjacent Links.*—Let Fig. 17 be a slider crank mechanism in which the

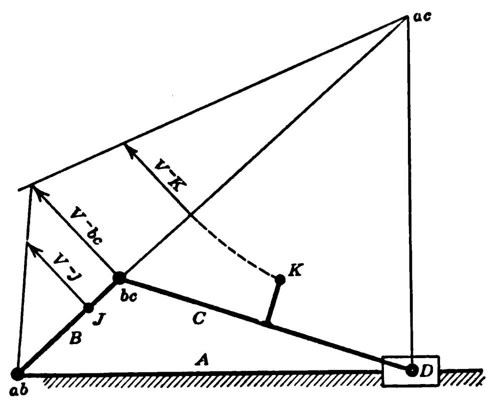


FIG. 17.

$V^{-}K$ is desired, having given the $V^{-}J$ with the link A stationary. The links to be considered are A, B and C, having centros ab, ac, and bc, of which ab and ac are stationary and bc is the common point.

Lay off a line representing the $V^{-}J$ and by similar triangles find the $V^{-}bc$. Then revolve K about ac to the line of centers and find its linear velocity.

Notice that the above figure represents the crank, connecting rod and crosshead of an ordinary steam engine; the frame and crosshead guides, being represented by the link A.

32. *Velocities in Non-adjacent Links.*—In the slider crank mechanism of Fig. 18 let the points J and K be in the links B and D respectively and the link A stationary.

The centros of the links A, B and D are ab, ad and bd. Revolve J about ab into the line of centers and lay off a line to represent its linear velocity, then find the $V^{-}bd$, the common point.

The stationary centro ad is at infinity, as it is the point in

A about which *D* rotates. The point *K* in being revolved about *ad* into the line of centers describes a straight line instead of an arc, or it is an arc of infinite radius. It will be noticed that the $V^{-}K$ is the same as the $V^{-}bd$, the common point.

The cases given above are but a few examples of the many that might arise. Remember that the velocities are the instantaneous

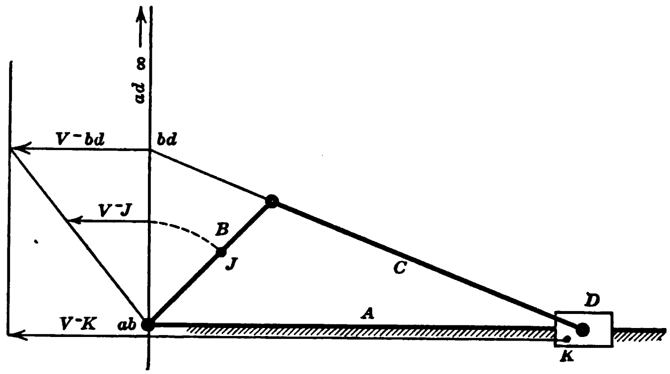


Fig. 18.

linear velocities, or velocities for the positions of the links shown. For any other position of the links the velocities would be different. Even though the linear velocity of the point *J* remained constant, the linear velocity of the point *K* would change for every position of the links.

PROBLEMS

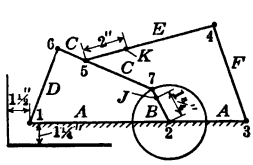
15. Take the link *D* stationary in Fig. 13 and assume the linear velocity of *J* in link *A*, to find the linear velocity of *K* in the link *C*.

16. Take the link *C* stationary in Fig. 16 and find the $V^{-}K$, assuming the $V^{-}J$.

17. Take the link *B* stationary in Fig. 17 and *J* in the link *A*. Assume the $V^{-}J$ and find the $V^{-}K$.

18. (*a*) Locate all of the centros.

(*b*) With the link *A* stationary and the link *B* making two revolutions per second, find the instantaneous linear velocity of *K* when the angle 1–2–7 = 60 degrees.



Data: Length $1\text{–}2 = 7\frac{7}{8}''$; $2\text{–}3 = 6\frac{1}{8}''$; $3\text{–}4 = 5\frac{7}{8}''$; $4\text{–}5 = 6\frac{5}{8}''$; $6\text{–}1 = 5\frac{1}{4}''$; $6\text{–}7 = 5\frac{1}{2}''$; $6\text{–}5 = 1\frac{1}{2}''$; $7\text{–}2 = 2''$.

Angle $1\text{–}2\text{–}7 = 60$ degrees.

Note.—Make a full size drawing using pencil lines only. Draw circles $\frac{3}{32}$ in. diameter at all pin joints. In plotting the velocity of J use a scale of 1 in. = 10 in. Put statement of problem in upper right-hand corner of sheet. Time, 2 hours.

CHAPTER IV

VELOCITY DIAGRAMS

33. Velocity Diagrams.—It is often desirable to find the velocity of a point for more than one instant, and this may be done by means of velocity triangles. There are two ways of representing the velocity, depending upon whether the point in question has a motion of translation or rotation, the former being shown by a *linear* velocity diagram and the latter by a *polar* velocity diagram.

34. Linear Velocity Diagram.—A linear velocity diagram is one in which the velocities are plotted by using rectangular coördinates.

In the several positions of the block A, shown in Fig. 19, let

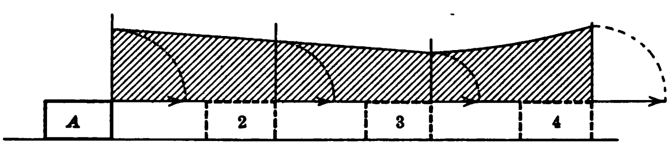


FIG. 19.

a length of line represent the instantaneous velocity of a point in the block, for each of these positions each line being drawn to the same scale. Revolve each velocity around the point, perpendicular to its direction of motion, and through each of the points cut off on the perpendiculars, draw a smooth curve, which is the velocity diagram.

To find the linear velocity of the block at any intermediate position, draw a line through the diagram perpendicular to the direction of motion, and the length of line cut off between the upper and lower sides of the shaded portion will be the linear velocity for that position.

35. Polar Velocity Diagram.—The polar velocity diagram differs from the linear velocity diagram in that polar coördinates are used instead of rectangular coördinates.

In Fig. 20 let A be a crank revolving about the center O, and let the velocity of the outer end in its various positions be represented by a length of line, each of these lengths being drawn to

the same scale. About the point in its several positions, revolve the velocity into the radius and draw a smooth curve through the points cut off on the radii.

The velocity at any intermediate point can then be assumed to be equal to the length of radial line cut off between the sides of the diagram.

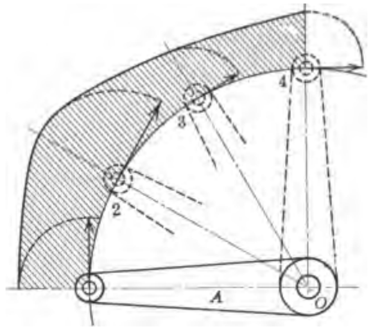


FIG. 20.

36. Velocity Diagram of Engine Crank and Crosshead.—In Fig. 21 let O be the crank shaft, OA the crank, AB the connecting rod, and B the cross head of an ordinary steam engine. If we know the length and revolutions per minute of the crank, its linear velocity can be found from the equation $V^{-}=2\pi \times OA \times n$ (Art. 14).

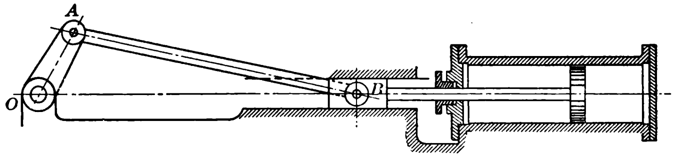


FIG. 21.

Then if the length of the connecting rod is also known the various instantaneous velocities of the crosshead can be found. The instantaneous velocities of the piston will be the same as those of the crosshead.

The angular velocity of the crank can be assumed to be constant throughout the revolution.

Draw the crank pin circle, Fig. 22, and divide it into any convenient number of parts as 12, corresponding to different crank positions. Find the extreme crosshead positions by taking O as

a center and radii, length of connecting rod plus length of crank, and length of connecting rod minus length of crank, giving M and N respectively. For the intermediate positions of the crosshead take a radius equal to the length of connecting rod and centers at 1, 2, 3, 4 . . . 12 on the crank pin circle and strike arcs cutting the center line of the crosshead in B, 2_1, 3_1, 4_1 and 5_1.

Lay off a length of line, drawn to any convenient scale to represent the V^- of the crank pin A, and revolve it around into the radius to get a point on the polar diagram.

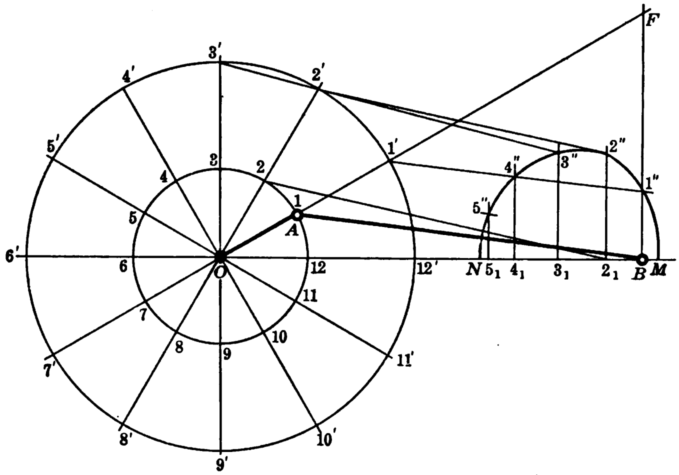


FIG. 22.

Since we assumed the angular velocity of the crank constant for all parts of the revolution, the linear velocity of any point will also be constant, and the polar diagram becomes a circle, the radius of which is the length of the crank plus the length of line representing the linear velocity of the crank pin. Knowing the direction of motion of each end of the connecting rod, its instantaneous center F can be found by drawing lines perpendicular to the direction of motion and their intersection will give the instantaneous center or centro (Art. 21).

The linear velocity of the crosshead end of the connecting rod will be to the linear velocity of the crank pin end directly as their instantaneous radii, or

$$\frac{V^-B}{V^-A}=\frac{BF}{AF} \text{ or } V^-B=\frac{BF}{AF}\times V^-A.$$

Instead of locating the instantaneous center of the connecting rod for each of its positions, we may draw through the various points on the polar diagram, 1′, 2′, 3′ . . . 12′, lines parallel to the connecting rod in its several positions, and the lengths of lines cut off on the perpendiculars drawn through the crosshead in its various positions gives the linear velocities of the crosshead $B-1''$, $2_1-2''$, etc. This is true because a plane passed through a triangle parallel to the base cuts off proportional parts from the other sides, and since the part cut off on one side will be the $V^{-}A$, the part cut off on the other side will be the $V^{-}B$.

After locating the points 1″, 2″, 3″ . . . 6″, draw a smooth curve through them. This curve will be the linear velocity diagram for the cross head. The curve above and below the line is symmetrical, so that it is not necessary to draw the lower portion.

37. In order to compare the linear velocity of the crosshead with that of the crank pin so that the difference in their velocities

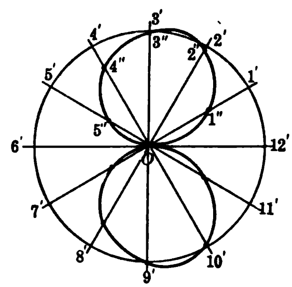


FIG. 23.

can be seen at a glance, they can be superimposed, one on the other.

In Fig. 23 draw a circle of radius equal to the linear velocity of the crank pin, and divide it into the same number of equal parts as the crank pin circle, in this case 12. Lay off on the first crank position, $O1''=B1''$ from Fig. 22 and on the second crank position, $O2''=2_12''$, etc., all the way around, then draw a smooth curve through the points 1″, 2″, 3″ . . . 12″ giving the velocity diagram of the crosshead. The distance 1″1′ is the amount of linear velocity of the crank pin in excess of that of the crosshead. In some parts of the revolution the linear velocity of the crosshead is greater than that of the crank pin. This occurs when the

instantaneous radius of the crosshead is greater than that of the crank pin.

38. Variable Motion Mechanism.—If the horizontal center line of the crosshead does not pass through the center of the crank as in Fig. 24, the linear velocity diagram for the crosshead will not be the same for the forward and back stroke. Let O be the center of the crank OA and MN the center line of the crosshead travel. The points M and N can be found by taking O as the center and radii equal to the sum and difference of the lengths of connecting

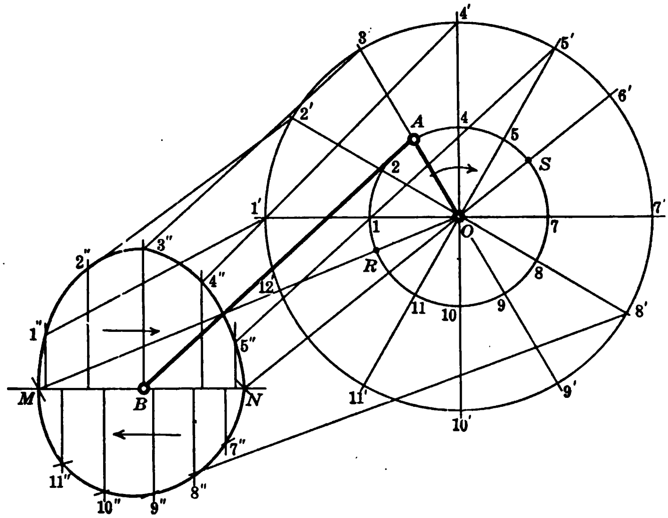


FIG. 24.

rod and crank respectively. Draw a line through M and O, cutting the crank pin circle at R, and through N and O, cutting the crank pin circle at S. R and S then, will be positions of the crank pin when the crosshead is at each end of its stroke.

If the crank moves in a clockwise direction as indicated by the arrow, the crosshead will move from M to N while the crank is going through the angle ROS which is less than 180°. The crosshead moves from N to M while the crank moves through the angle SOR which is more than 180°, so that if the linear velocity of the crank pin is constant throughout the revolution, the crosshead will have to move from M to N at a faster rate than it does when travelling from N to M, and this is shown by the linear velocity diagram of the crosshead, the area of the

curve above the horizontal line MN being greater than the area of the curve below the line.

The construction of the velocity diagram is similar to that of Fig. 22.

39. By using the same combination of links as shown in Fig. 22, but by making what is there the crank stationary, and using the frame as one of the moving links and also changing the proportions of the links another variable motion mechanism can be obtained.

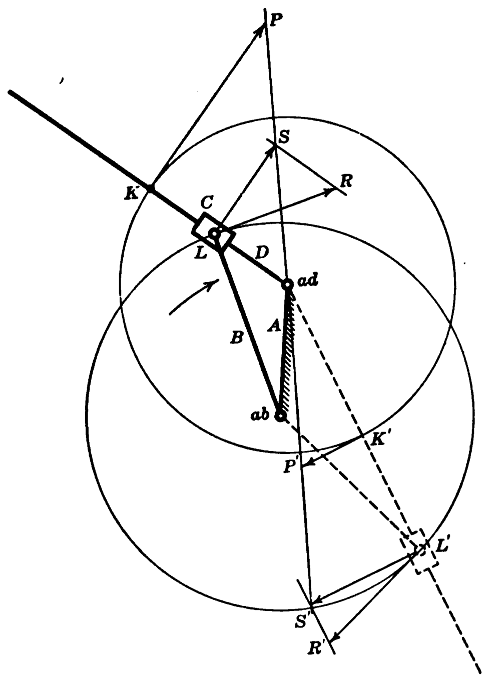


Fig. 25.

Let A, B, C and D of Fig. 25 be the links, with A stationary. Assume that the end of the revolving arm B, which is fastened to the sliding block, has a constant velocity, represented by the length of line LR, laid off at right angles to B. To find the linear velocity of K, a point in the link D, when the mechanism is in the position shown by the full lines.

The velocity of K can be found by the method of instantaneous centers as was done in a previous chapter, but a more simple

way where the velocity is to be found for a number of different positions is as follows.

The length of line *LR* represents the velocity of the sliding block, turning about the center *ab*. To find the velocity of *LR* turning about the center *ad*, resolve *LR* into two components, one *LS* at *right angles* to *D*, and the other *RS parallel* to *D*. The length of line *LS* represents the velocity at which the block is turning about the center *ad*, while *RS*, is its sliding velocity along *D*. Knowing the turning velocity of the sliding block around the center *ad*, we can find the turning velocity of *K* around the same center by means of similar triangles, by drawing the line *ad–P* through *S*, and *KP* parallel to *LS*. The length of line *KP*, then represents the linear velocity of *K* to the same scale that *LR* represents the linear velocity of the end of *B*, since linear velocities of points in the same link are directly proportional to their radii. In the dotted positions of the links is shown the velocity of *K* for the same length of line representing the linear velocity of *L*.

40. The link *B* is called the *constant radius arm* since its length does not change and the center about which it rotates is called the *constant radius arm center*. The link *D* is called the *variable radius arm*, since its radius from the sliding block to the center *ad* is continually changing. Its center is called the *variable radius arm center*.

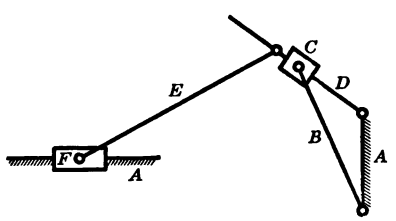


FIG. 26.

41. Whitworth Quick Return Motion Mechanism.—By attaching a connecting rod to the link *D*, Fig. 25, and a crosshead, or ram to the connecting rod, a mechanism, known as the Whitworth Quick Return Motion is obtained. It is shown in simple diagrammatic form in Fig. 26, while Fig. 27 shows a form used in machine tools.

The method of laying out the velocity diagrams can best be shown by working out a problem.

Problem.—Design a Whitworth Quick Return Motion Mechanism for a shaper, and construct the velocity diagram of the ram for the maximum stroke.

Velocity ratio forward to return stroke 2:1.

Distance between constant radius arm center and variable radius arm center 2 in.

Center of variable radius arm center above center line of ram $2\frac{1}{2}$ in.

Length of stroke—maximum, $5\frac{1}{2}$ in.

Length of stroke—minimum, 4 in.

Length of connecting rod, 12 in.

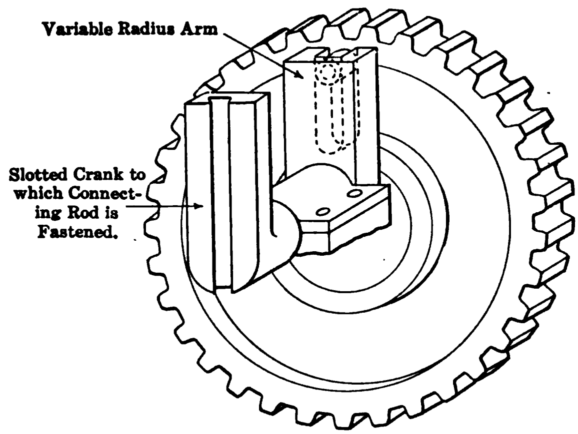


Fig. 27.

Layout the maximum stroke AB, Fig. 28, and draw the horizontal line CD at a distance above AB equal to the height of the variable radius arm center above the center line of the ram.

With A as a center and a radius equal to the length of the connecting rod, draw the arc mn, and with the same radius and center B, draw the arc rs.

Locate O, the center of the variable radius arm by trial, on the line CD, by taking O as the center of a circle, the circumference of which will be tangent to the arcs mn and rs.

Through A and O draw a line intersecting the arc mn at K, and one through B and O intersecting the arc rs at K'. Then OK and OK' are the positions of the variable radius arm when the ram is at the extreme ends of its stroke A and B respectively.

Draw a line KK' and through O, perpendicular to KK' draw the line OG of indefinite length. Locate the constant radius arm center G on this line at the given distance from O, the variable radius arm center.

Next locate the positions of the constant radius arm when the ram is at each end of its stroke.

In this problem the velocity ratio is 2:1 or it takes twice as long for the forward stroke as the return stroke, and since the constant radius arm revolves at a constant velocity, it will take 240° of revolution for the forward stroke and 120° for the return stroke.

With G as a center lay off two lines symmetrical with OG, making an angle of 120° with each other. These lines will inter-

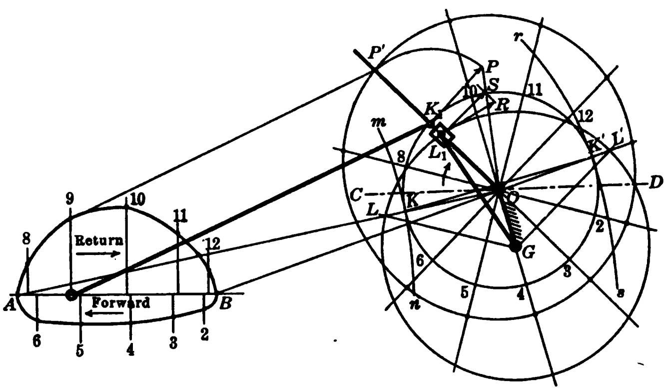


FIG. 28.

sect the lines through OA and BO at L and L' respectively, giving the positions of the constant radius arm when the ram is at the ends of its stroke.

With G as a center and radius GL describe a circle, which will be the path of the end of the constant radius arm.

Divide the variable radius arm circle up into any convenient number of equal parts, as 12, symmetrical with the line OG, to represent different positions of the variable radius arm and find the corresponding positions of the ram by taking centers at 1, 2, 3 . . . 12, and a radius equal to the length of the connecting rod and striking arcs cutting the line AB.

Since the velocity of the constant radius arm is constant for all parts of the revolution we may represent it by a line of any

convenient length. Let L_1R represent the velocity of the end of the constant radius arm, laid off at right angles to the radius. To find the V^-K, resolve L_1R into two components (Art. 39), one L_1S perpendicular to the variable radius arm K_1O and the other RS, parallel with it. The component L_1S is the linear velocity with which L_1 is turning about the center O, and the V^-K_1 which is turning about the same center, is found by similar triangles and is equal to K_1P. Revolve P around to P' to get a point on the polar velocity diagram and through P' draw a line parallel to the connecting rod, getting the point 9 on the linear velocity diagram of the ram.

Repeat the process for each of the other points by using the same length of line to represent the linear velocity of the constant radius arm, resolve it into its two components, find the velocity of the joint of the connecting rod and variable radius arm, and from that the velocity of the ram.

After completing the diagram it will be noticed that the velocity for the forward stroke of the ram is very much more constant than for the return stroke.

The minimum stroke of the ram is obtained by moving the end of the connecting rod K_1, toward O until the ram moves through the required minimum stroke. The radius OK_1 will not however, be one-half of the stroke because O does not lie on the line AB.

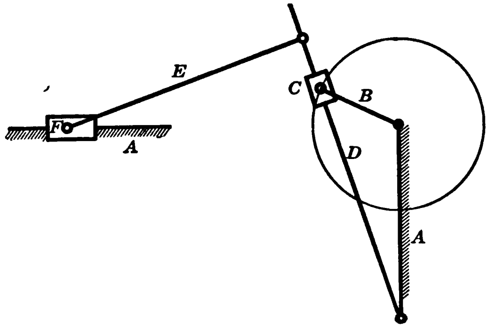


Fig. 29.

It will be seen from the figures illustrating this mechanism that the constant radius arm and the variable radius arm, both describe complete circles.

42. Oscillating Arm Quick Return Motion Mechanism.—By using the same combination of links as in the Whitworth Quick

Return Motion Mechanism, but with different proportions, a mechanism known as the Oscillating Arm Quick Return Motion can be obtained.

Let Fig. 29 represent it in the same manner that Fig. 26 represented the Whitworth Quick Return Motion. Note that the distance between the fixed centers is much greater and that the variable radius arm D is much longer than the constant radius arm, so that while the constant radius arm revolves in a complete circle, the variable radius arm oscillates. The angle through through which it oscillates is found by drawing lines from the variable radius arm center, tangent to the circle described by the end of the constant radius arm.

Except for the differences noted, the method of laying out the velocity diagram of the ram is similar to that of the Whitworth Quick Return Motion.

PROBLEMS

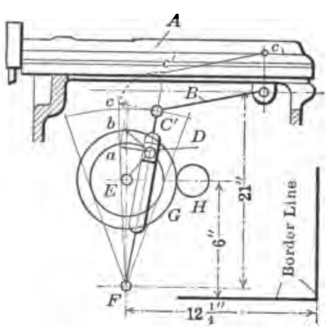
19. Make sketches showing the essential difference between the Whitworth quick-return mechanism and the oscillating arm quick-return mechanism.

20. Is the velocity diagram of the crosshead, Fig. 22, symmetrical about a vertical axis? Why?

21. Make a sketch showing how the velocity diagram for the crosshead is obtained when its path of travel is above the center of the crank.

22. (*a*) Lay out the velocity diagram of the ram for the oscillating arm shaper.

(*b*) How many r.p.m. must H make in order that the maximum cutting speed be 60 ft. per minute? Diameter of $G=12''$. Diameter of $H=4''$.



Data: Length of oscillating arm $C=20''$. Length of link $B=9\frac{1}{2}''$; maximum length $DE=4\frac{1}{4}''$. Time ratio (maximum stroke) $=5:3$.

Note.—Make a skeleton pencil drawing, half size. Lay off the line representing the velocity of $D = \frac{DE}{3}$ (not half size).

Put no statement of problem or data on sheet, but this title:

LAYOUT OF VELOCITY DIAGRAMS

FOR

OSCILLATING ARM QUICK-RETURN MECHANISM

Time, 8 hours.

CHAPTER V

PARALLEL AND STRAIGHT-LINE MOTION MECHANISMS

43. A parallel motion mechanism is a combination of links so constructed that when one point in it moves in any path, another point will move in a parallel path.

Parallelogram.—The parallelogram is one of the most simple mechanisms for giving an exact parallel motion. It consists of four links, the lengths of opposite ones being equal.

Fig. 30 is a parallelogram in which D, the joint of the two links CD and AD is fixed.

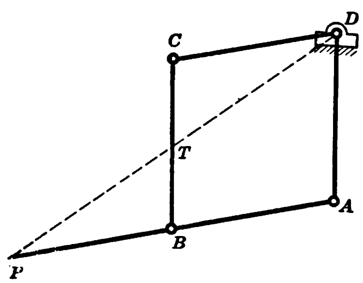


FIG. 30.

Take a point P on the link AB extended and draw the line PD, cutting the link BC at T, then in the triangles PBT and PAD.

$\frac{BT}{AD} = \frac{PB}{PA}$ or $BT = \frac{PB}{PA} \times AD =$ a constant.

Therefore T is always at the same point on BC for a given position of P on the link AB.

Also, $\frac{DP}{DT} = \frac{AP}{AB}$, which is constant for all positions of the linkage so that if T is the tracing point and is made to follow any outline, the pencil point P, will trace a similar outline, only on a larger scale. If the link BC is extended and T taken on the extension as in Fig. 31, P will fall between D and T, and P will then trace an outline smaller than that traced by T.

By moving T along the link BC, any ratio of size can be obtained the ratio being $\frac{DP}{DT}$.

It will be noticed in these figures that the fixed, pencil and tracing points lie on the same straight line. This must always be true.

Fig. 32 shows the fixed point O on an extension of one of the links, while in Fig. 33 it is between the ends of the links. The

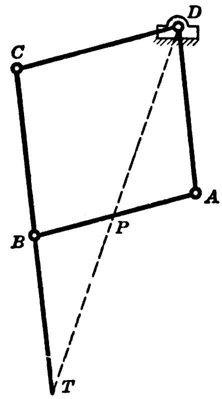


FIG. 31.

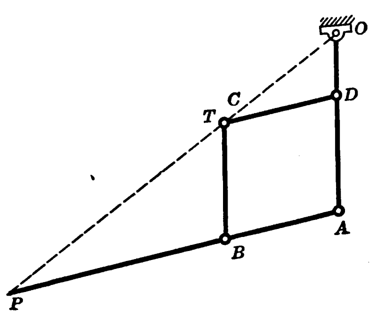


FIG. 32.

same principles hold good for these cases as in the first two considered.

In Fig. 33 the ratio $\frac{OP}{OT} = 1$ so that P traces an exact duplicate of that traced by T.

These mechanisms are used for copying and enlarging maps, engravings, etc.

In the latter case the pattern is usually made to an enlarged scale to eliminate errors, the engraving tool being an end

milling cutter revolving at a high speed, is made to produce the similar pattern reduced or enlarged as the case may be.[1]

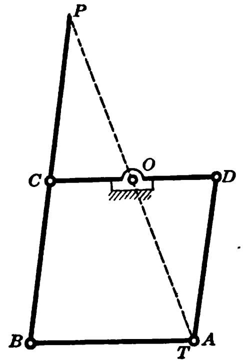


Fig. 33.

44. Parallel Ruler.—Fig. 34 shows the application of the parallelogram to the parallel ruler for drawing parallel lines.

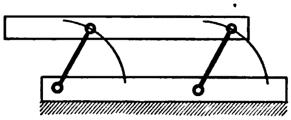


Fig. 34.

45. Roberval Balance.—By adding a fifth link to the parallelogram, a double parallelogram is obtained in which an application is found in the druggists' balance of Fig. 35. By placing the

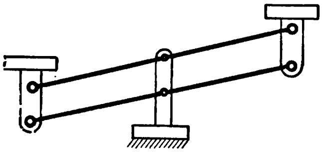


Fig. 35.

support midway between the scale pan uprights, the load will be the same on each pan when they are at the same level.

46. Drafting Board Parallel Mechanisms.—There are many of these devices for guiding the straight-edge with a parallel motion, only a few of which will be shown here.

[1] For a description of an engraving machine see *Machinery* of April, 1911, page 602.

In Figs. 36 and 37 the straight-edge is made so that it projects beneath the board at the ends, as well as being on top.

In Fig. 36 one cord is fastened to the straight edge where it

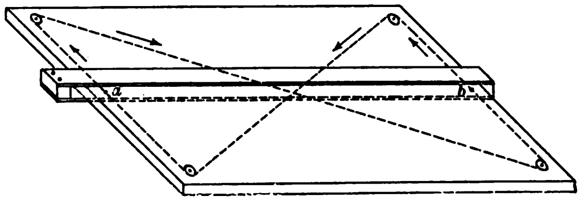


FIG. 36.

projects beneath the board at a, carried up and around one pulley, then diagonally down across the board around a second pulley and fastened to the straight edge at b. Another cord

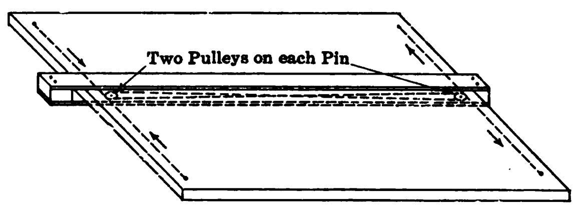


FIG. 37.

beginning at b is carried up and around a pulley then diagonally across the board around a second pulley and up to a. Fig. 37 is somewhat similar except that there are two pulleys each at a and b and the cords are fastened to the corners of the board.

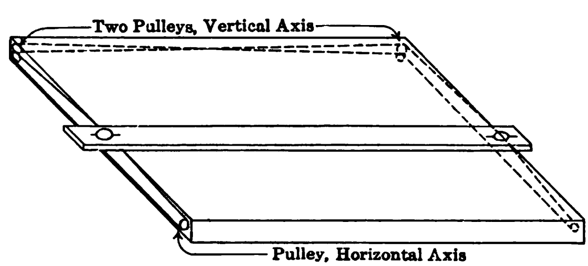


FIG. 38.

In Fig. 38, six pulleys are required, but the cord is continuous. Fig. 39 shows diagrammatically the "Universal Drafting Machine." This machine takes the place of the tee-square,

scales, triangles and protractor. The mechanism consists of two sets of parallelograms, so arranged that in no matter what position they are placed, the scales will be parallel to their original positions. The head to which the scales are connected is movable about its center, thus allowing any angle of the scales with the horizontal, the most common angles being

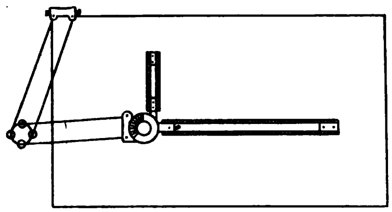


FIG. 39.

obtained by a pin dropping into holes, and the others by means of a vernier and the head clamped by means of a thumb screw.

47. Straight-line Motion Mechanisms.—A straight-line motion mechanism is one that will cause some point in the mechanism to move in a straight line without the aid of guides.

It was necessary to use them originally when it was desired to make some part move in a straight line, as for example, the

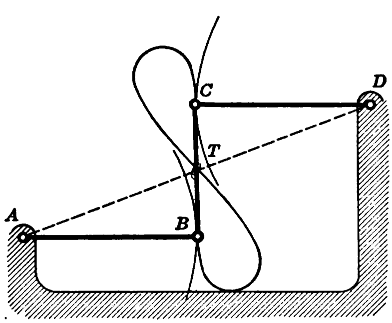


FIG. 40.

crosshead of an engine, because machine tools were not far enough developed to make plane surfaces.

Most of the so-called straight-line motions are only approximate, while there are a few that are mathematically correct.

48. Watt's Straight-line Motion.—This is the best known and most widely used of all the straight-line motions. In Fig. 40,

it is shown in its simplest form. In the figure the connecting link *BC* is shown perpendicular to *AB* and *CD* when they are parallel, but while this is the best proportion, it is not necessary that they be so. The tracing point *T* is located on *BC*, where a line joining *A* and *D* intersects it.

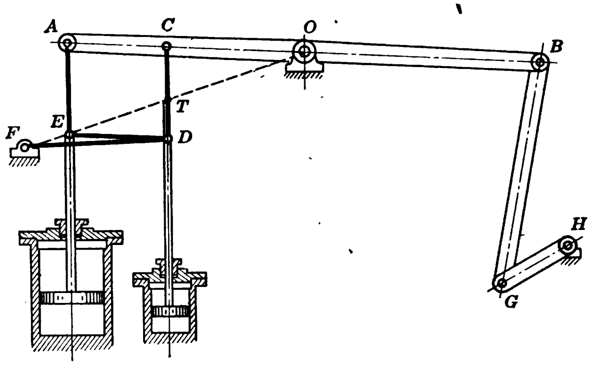


Fig. 41.

Fig. 41 shows the application of the Watt straight-line motion in combination with the form of parallelogram shown in Fig. 32, to a walking beam engine. *O* is the center of the walking beam, *F*, a point on the frame and *H*, the crank shaft bearing. The links *OC*, *CD* and *DF* compose the Watt motion mechanism, the point *T*, moving in a straight vertical line, while the links *OA*, *AE*, *ED* and *DC* make up the elements of the parallelogram.

The point *T* is found as in Fig. 40 and *E* is taken on a line

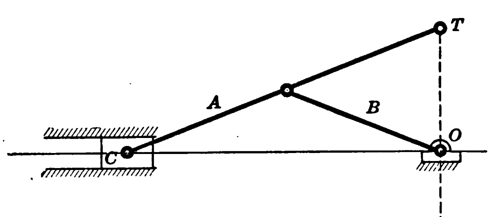


Fig. 42.

drawn through *O* and *T*. In practice *OC* and *FD* are usually taken as one-half of *OA*. A pump rod is usually attached to link *CD* at *T* as shown.

49. Scott Russell Straight-line Motions.—In this mechanism shown in Fig. 42 there are two links *A* and *B* and the sliding

block *C*. *B* is one-half the length of *A* and is fastened to *A* at its middle point. Take a center at the intersection of *A* and *B* and a radius of length *B;* strike an arc which will pass through the points *T*, *O* and *C*, so that the angle *TOC* is a right angle, and *T* will move in a straight line. This is an exact straight-line motion, but it requires the sliding block moving between plane guides.

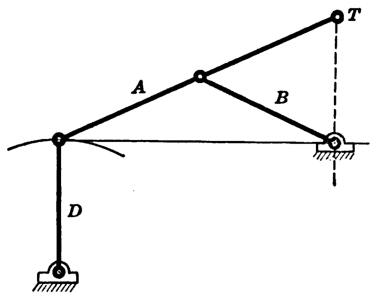


Fig. 43.

Fig. 43 shows a modification of the mechanism, in which the link *D* is substituted for the sliding block. In this case, the longer the arm *D*, the more nearly will *T* move in a straight line. This is sometimes known as the "grass-hopper motion."

In Fig. 44 is shown a second modification of the mechanism that will give an approximate straight line. In this case, the arm *E* is substituted for *D* of the previous figure.

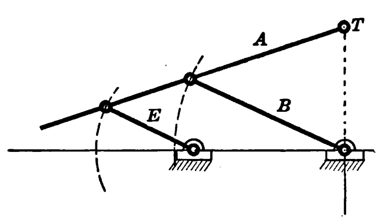


Fig. 44.

50. Thompson Indicator Motion.—Fig. 45 shows the application of the Scott Russell motion to a Thompson steam engine indicator.[1] The pencil at *T*, which traces the diagram on an

[1] For various other forms of indicator mechanisms, together with the proportions as made by the various manufacturers, see "Machine Design," Part 1, by Forrest R. Jones.

oscillating drum, is guided by a Scott Russell straight-line motion, consisting of the links OA, AT and CD. The length of link BE is determined by drawing a line through O and T, and noting the point E, where it cuts the center line of the cylinder. If instead of the link CD, the link FE were used, the linkage would then be the parallelogram shown in Fig. 32. This link

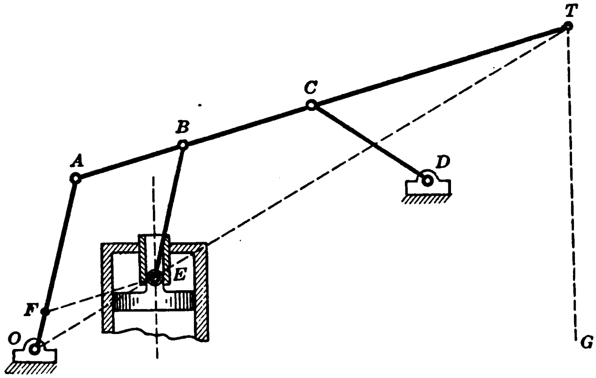


FIG. 45.

is impossible however as it would cut the side of the steam cylinder of the indicator.

While the pencil point does not move in an exact straight line TG, the error is slight.

51. Tchebicheff's Approximate Straight-line Motion.—This mechanism devised by Prof. Tchebicheff of St. Petersburg, is

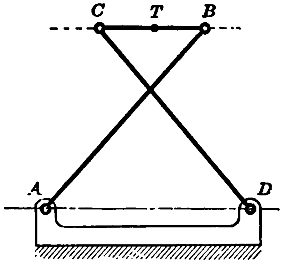


FIG. 46.

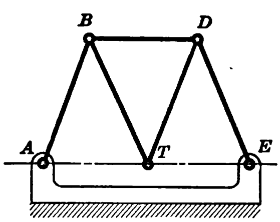


FIG. 47.

shown in Fig. 46. The proportions of the links are such that $AB = CD = 5$, $BC = 2$ and $AD = 4$. The tracing point T in the middle of BC moves in an approximate straight line parallel to AD.

52. Robert's Approximate Straight-line motion.—Fig. 47 shows this mechanism which consists of two equal links AB and DE and a triangular shaped link BTD in which $BT=TD=AB$, and $BD=\frac{AE}{2}$. The tracing point T, closely follows the straight line AE.

53. Peaucellier's Exact Straight-line Motion.—This mechanism Fig. 48, consists of seven movable links and one stationary link. The proportions are such that $AO=BO$; $AC=BC=BT=AT$,

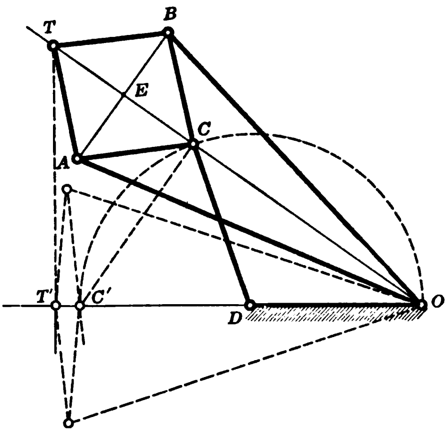


Fig. 48.

and $CD=DO$. Then O, C and T will always lie on the same straight line.

and $\overline{OB}^2-\overline{OE}^2=\overline{BT}^2-\overline{TE}^2$

or transposing $\overline{OB}^2-\overline{BT}^2=\overline{OE}^2-\overline{TE}^2$

$$=(OE+TE)(OE-TE)$$
$$=OT\times OC$$

Therefore $OT\times OC$ is a constant.

Now suppose that the linkage is moved until C falls at C' and let T' be the new position of T.

Then we have $OC'\times OT'=OC\times OT$

$$\text{or } \frac{OC'}{OC}=\frac{OT}{OT'}$$

And $OD=DC=DC'$ so that O, C and C' lie on a semi-circle which makes the angle OCC' a right angle.

In the triangles COC' and TOT', the angle TOT' is common, and since the sides are proportional, the triangles are similar.

Thus since the angle OCC' is a right angle, the angle $TT'O$ is also a right angle.

In the same way it can be shown for any other position of the linkage, that the line TT' is perpendicular to OT', and therefore T moves in a straight line.

In applying this motion to engines, the piston rod is fastened to T and this takes the place of the usual crosshead and guides.

If the link DC is not made equal to DO, T will describe a circular arc.[1]

If $\frac{CD}{OD} < 1$, the arc described by T will be concave toward O, while if $\frac{CD}{OD} > 1$, the arc will be convex toward O.

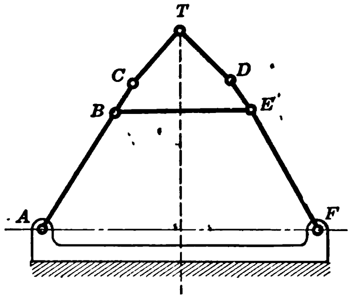


FIG. 49.

54. Bricard's Exact Straight-line Motion.—If the linkage of Fig. 49 is made of the proportions[2] such that $AC = DF = a$; $CT = TD = b$; $AF = c$; $BC = DE = \frac{b^2}{a}$, and $BE = \frac{bc}{a}$, the point T will describe a straight line perpendicular to and bisecting AF.

PROBLEM

23. With the arrangement of the links of a parallelogram as shown in Fig. 30, let $DA = 10$ in.; $DC = 6$ in.; $CT = 4$ in. Will the copy be larger or smaller than the original and how much?

[1] For the proof of this see "Kinematics of Machines," by R. J. Durley.

[2] For these proportions and the proof see Prof. Durley's "Kinematics of Machines," page 94.

CHAPTER VI

CAMS

55. A cam is commonly a plate or cylinder that transmits motion to its follower by means of its edge or a groove cut in its surface.

The cam plays a very important part in the construction of many machines, among which are sewing machines, shoe and printing machinery. Their shapes are as numerous as the uses to which they are put, so that only some of the most common forms will be considered.

56. Contact Between Cam and Follower.—Theoretically the contact between a cam and its follower is a *point* or a *line*, see Fig. 50. The disadvantage of these can readily be seen, since

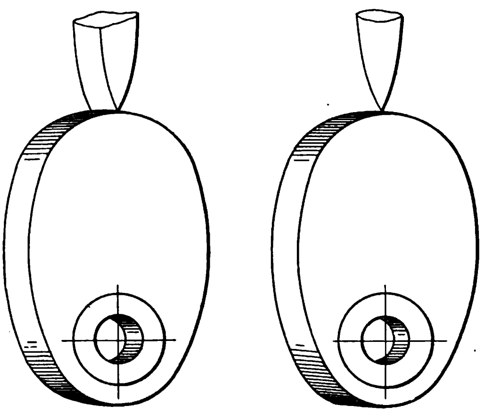


Fig. 50.

the wear would be excessive, so that for practical purposes a roll is generally substituted, as in Fig. 51. In this case the roll turns on a pin that is rigidly fastened to the follower, so that while there is sliding contact between the pin and the roll, there is rolling contact between the roll and the cam. This has the advantage that nearly all of the wear is concentrated on the pin which can be replaced easily.

The ordinary diameters of rolls are from ½ in. to 2 in. diameter. Sometimes a flat face follower as shown in Fig. 52 is used. This type of follower is better adapted for use with light loads and a small rise, than for heavy loads and a large rise.

57. Base Circle.—The base circle is a circle with its center at the center of the cam shaft and a radius equal to the shortest distance to the theoretical cam curve.

It is from the base circle that the cam curve is laid out, and

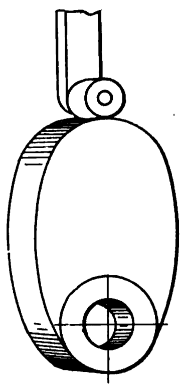


Fig. 51.

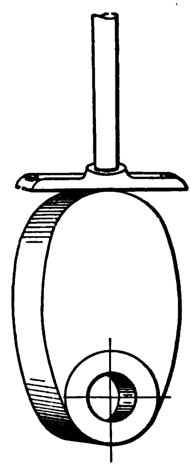


Fig. 52.

the base circle should be of such a size that will insure an easy motion to the follower.

The larger the base circle, the easier the motion will be for any given rise in a given angle as is shown in Fig. 53. Let O be the center of the cam shaft and the angle through which the rise is to take place be 30°. Let $1-2=2-3=3-4=4-5=$ the rise that is to be given the follower in 30° when using base circles of radii $O1$, $O2$, $O3$, and $O4$. It can be seen that the line IV representing the rise when using the base circle of radius $O4$ is the most gradual slope of any of the lines.

On the other hand, the larger, the base circle, the larger the resulting cam will be, and the designer often has to make the cam small in order to get it into a certain space, so that it is largely a matter of judgment as to how large to make the base circle, but it should be large enough to leave plenty of stock around the cam shaft.

When using a small base circle and a large rise of follower in a small angle of revolution of cam, a difficulty that is apt to be encountered is shown in Fig. 54. The theoretical curve is shown by the dot two dash line, and the working curve is found, if using a roll follower by taking centers of the roll on the theoretical curve and striking arcs, then drawing a smooth curve tangent to these arcs. This curve is the working curve. At the top of the cam shown in the figure, the distance from the center of the roll to the working curve, represented by A is more than the radius of the roll, so that the cam would not be raised to the proper height by the difference between the radius of the roll and the distance A.

The remedy for this is to use a larger base circle or a smaller roll or both.

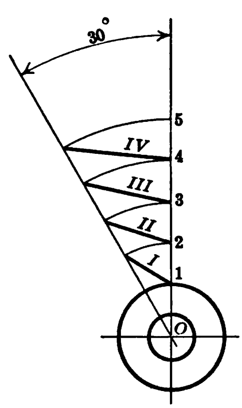


FIG. 53.

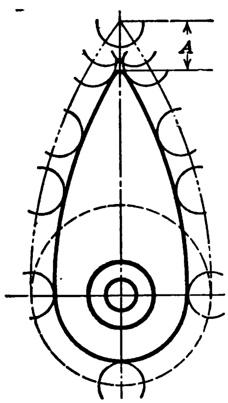


FIG. 54.

58. Motions used for Cam Curves.—The motions most commonly used in the layout of cams are *uniform*, *harmonic*, *uniformly accelerated* and *uniformly retarded.*

59. Uniform Motion.—As applied to the motion of the cam follower, uniform motion means equal rises of follower in equal intervals of time, the time being measured by divisions or degrees on the base circle.

The heart-shaped cam of Fig. 55 is an example of this kind.

In this cam the base circle has a radius OA and the follower is to be raised a distance $O8$ in 180° of revolution of the cam with a uniform motion, and to drop a distance $O8$ in 180° of revolution with a uniform motion.

The method of laying out the curve is as follows:

Divide the semi-circumference of the base circle in any number of equal parts, in this case 8, and after laying out the rise $O8$ on any of the radial lines, divide it into the same number of equal parts as the semi-circumference of the base circle, and number the points on the rise to correspond to the divisions into which the base circle is divided.

The radial lines dividing the base circle can be assumed to be the different positions of the center line of the follower revolving around the cam, as it is not possible to revolve the cam on the paper. With a center O and radius $O1$ strike an arc intersecting

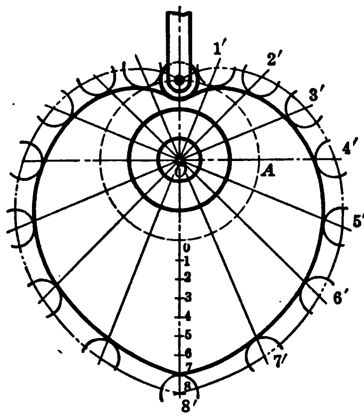


FIG. 55.

$O1'$ at $1'$; with the same center and a radius $O2$ intersect $O2'$ at $2'$. Do this for all the divisions of the rise, then draw a smooth curve through the points $1', 2', 3' \ . \ . \ . \ 8'$ which will give the *theoretical curve* for one-half of the cam. Since the second half of the cam is the same, if the second half of the base circle is divided into the same number of equal parts as the first half, the same points on the radial line can be used for finding the points on the theoretical curve.

If the follower were of the knife edge type shown in Fig. 50, the curve just found would also be the *working curve*, but where a roll follower is used, it is necessary to find the working curve by taking a radius equal to the radius of the roll, and with centers on the theoretical curve strike arcs, then draw a smooth curve tangent to these arcs which will give the working curve.

It will be seen from this, that for the same theoretical curve, an infinite number of working curves can be obtained by changing the diameter of the roll.

60. Harmonic Motion.—The projection on the diameter of a circle of a point moving with uniform velocity in the circumference is said to have harmonic motion. The application of this motion to the follower of a cam can perhaps best be seen by working out a simple problem.

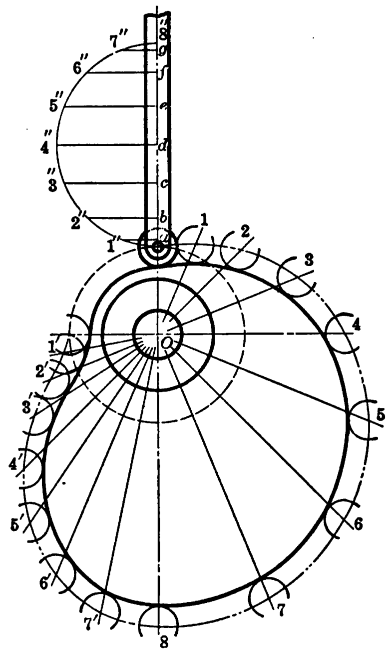


Fig. 56.

Problem.—Design a disk edge cam that will give a reciprocating follower a harmonic rise of 4 in. in 180° of the cam's revolution, a harmonic drop of 4 in. in the next 90°, and a "rest" or "dwell" during the remaining angle. Cam to revolve counter clockwise.

Layout the base circle of radius OO, Fig. 56, and divide the 180° angle through which the cam is to revolve during the harmonic rise of the follower into any convenient number of equal parts, as 8, and draw radial lines through these points. On one of these radial lines lay off 08″ from the base circle equal to 4 in. Take 08″ as a diameter and construct a semi-circle. Divide

the circumference of the semi-circle into the same number of equal parts as the 180° of revolution, thus getting the points 1″, 2″, 3″ . . . 8″. Through the points 1″, 2″, 3″ . . . 8″ drop perpendiculars to the diameter of the semi-circle to get the points a, b, c . . . g. The distances $0a$, ab, bc, etc., will be the rise of the follower for equal angles through which the cam revolves. With O as a center and radii Oa, Ob, Oc . . . $O8''$, strike arcs cutting $O1$, $O2$, $O3$, etc., at 1, 2, 3, etc. Draw a smooth curve through 1, 2, 3, etc., which will be the theoretical curve.

The next angle through which the cam turns for the 4 in. harmonic drop is 90°. Divide the 90° into any convenient number of equal parts. Since the drop is the same amount, as the rise, if the 90° is divided into the same number of equal parts as the rise, the same construction for the harmonic motion can be used.

During the remaining angle the follower is to remain at "rest" which means that during that part of the revolution the follower does not move toward or away from the center of the cam, so that the theoretical curve is the arc of a circle with a center at O and a radius $O0$, which in this case is the radius of the basc circle. The working curve is found in the same way as was done in the previous case.

61. Uniformly Accelerated Motion.—This is motion with a uniformly increasing velocity. It is the easiest motion that can be given to the follower of a cam.

When the follower starts from rest, the formula $S=\frac{1}{2}aT^2$ is applicable, where S = space passed over; a = acceleration (a constant) and T = time.

Taking the successive angles through which the cam turns as the units of time, the space passed over in

1 unit of time, $S=\frac{1}{2}a=k$ (a constant).
2 units of time, $S=2a=4k$.
3 units of time, $S=4\frac{1}{2}a=9k$.
4 units of time, $S=8a=16k$.
5 units of time, $S=12\frac{1}{2}a=25k$.

So that if the angle through which the cam turns, and the total rise of the follower during which it has uniformly accelerated motion are known, by dividing the angle up into any number of units, we can solve for k

For example, if the total rise is 3 in. and there are five units of time in the angle, $k=\frac{3}{25}=0.12$ in. Lay off from the base circle on the line at the end of the first unit of time 0.12 in. and on the second line 0.12 in.$\times 4=0.48$ in., on the third 0.12 in.$\times 9=1.08$ in., on the fourth 0.12 in.$\times 16=1.92$ in., and on the fifth line 0.12 in.$\times 25=3$ in. These are the various distances that the follower has been moved away from the base circle at the different intervals of time.

Instead of calculating this amount each time, a graphical method is often used and is much more simple.

It will be seen that in the series $1k$–$4k$–$9k$–$16k$–$25k$, etc., the distance that the point has moved from the starting place at the end of successive units of time, that the distance moved during each successive unit of time is in the series 1–3–5–7–9, etc.

In order to apply this to the follower of a cam, a simple example will be used.

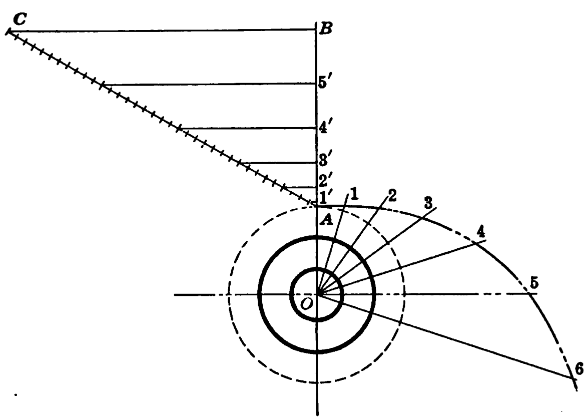


FIG. 57.

Let AB, Fig. 57, be the distance that a reciprocating follower is to be raised with a uniformly accelerated motion while the cam turns through the angle $AO6$. Divide the angle $AO6$ into any convenient number of equal parts as six, and on a line AC of indefinite length lay off units in the series 1–3–5–7–9–11, using as many numbers in the series as there are divisions in the angle, in this case six. Through the end division and through B draw a line; parallel to this line and through the other points on AC draw lines intersecting AB at $5', 4', 3', 2'$ and $1'$. This gives divisions on AB that are proportional to those on AC.

With O as a center and radii $O1'$, $O2'$ . . . OB strike arcs intersecting $O1$, $O2$ . . . $O6$ at 1, 2, 3 . . . 6. Through these latter points draw a smooth curve, which will be the theoretical cam curve.

62. Uniformly Retarded Motion.—This is the opposite of uniformly accelerated motion, and when applied to a cam curve, the series is laid off in the reverse order as 11–9–7–5–3–1. Very often both motions are combined so that the follower starts slowly and is uniformly accelerated for half of the rise and uniformly retarded for the second half, thus bringing it to rest slowly. In using the two the angle must always be divided into an *even* number of equal divisions, as there will be the same number accelerated as retarded. If there were eight equal divisions, the series would be 1–3–5–7–7–5–3–1, and should be laid out in the same way as was described for uniformly accelerated motion.

63. Cams with Off-set Followers.—In the cases considered thus far, the center line of the follower has been taken directly over the center of the cam shaft, but there are numerous cases where the follower is "off-set" and while the principle of laying out the curve is the same there are some minor differences which will be illustrated by a problem.

Problem.—Design a disk-edge cam that will raise a reciprocating roll follower 3 in. with harmonic motion in 90° of the cam's revolution; the follower to remain at rest for 90°, and to drop with uniform motion in the remaining angle. Center line of follower $\frac{3}{4}$ in. to left of cam shaft center. Cam to turn clockwise.

Draw a base circle, Fig. 58, of any convenient radius and $\frac{3}{4}$ in. to the left of center of cam draw the center line of the follower, AB. With O as a center and a $\frac{3}{4}$ in. radius draw a circle. Divide the base circle by starting at the point where the center line of the follower cuts it, and draw lines through the divisions on the base circle and tangent to the circle of $\frac{3}{4}$ in. radius, which has already been drawn. These lines, as in the previous cases, represent different positions of the center line of the follower, as if it were revolved around instead of the cam turning.

Since the first angle is 90°, the last division will be horizontal, as the first one is vertical. Divide the 90° into any convenient number of equal parts as six, and lay off on AB from its intersec-

tion with the base circle, 3 in. the rise of follower. With 3 in. as a diameter draw the semi-circle, and divide the semi-circumference into six equal parts 1′, 2′, 3′, 4′, 5′ and 6_1. Project these points on the diameter, giving the points 1_1, 2_1, 3_1, 4_1, 5_1 and 6_1. With O as a center and a radius $O1_1$, strike an arc cutting the first division of time at 1, and so on for all of the points, always using the center O. Since the follower is to remain at rest during the next 90°, the theoretical cam curve will be the arc of a circle for 90°, with O as a center and a radius $O6_1$.

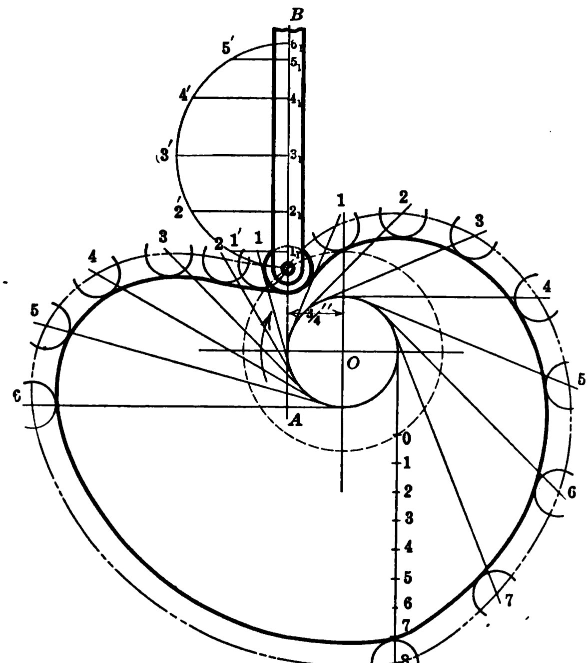


FIG. 58.

The cam has now turned through 180° which leaves 180° to complete the revolution. Divide the 180° into any convenient number of equal parts, as eight, and as the drop is to be uniform, also divide the 3 in. into eight equal parts. Then with O as a center, find points on the theoretical curve as before.

64. The Flat Face Follower.—In any of the foregoing cams, the flat face follower can be substituted for the roll follower and the method of laying out the cam is exactly the same up to the point of laying out the working curve.

Using the same data as was used in Fig. 58, let us design the cam for a flat face follower, the face of the follower being perpendicular to its center line.

Let us assume that the theoretical curve has already been found. To get the working curve draw lines 1*a*, 2*b*, 3*c*, etc., Fig. 59, perpendicular to the center line of the follower in its various positions. After these lines are drawn in they will form a series of triangles, which are shown cross-hatched in the figure. Draw a smooth curve tangent to the middle point of each of these triangles, which will be the working curve.

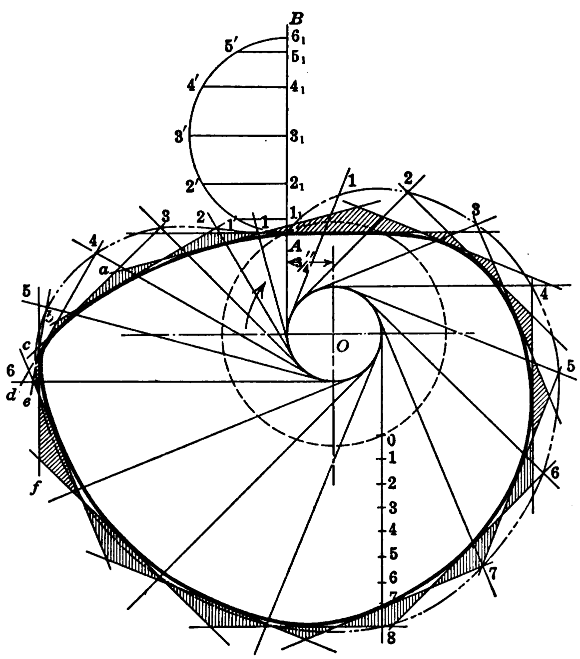


FIG. 59.

If it is impossible to draw the curve tangent to all of the lines without crossing some of the others, the curve cannot be found, and a larger base circle must be taken.

The length of the flat face of the follower is found by taking the sum of the two greatest distances from the theoretical curve on the center line of the follower out on the perpendicular lines to where the working curve is tangent. Take one of these on the rising part of the curve and the other on the falling side.

These distances will be the lengths of the follower face required on the left hand and right hand side of the center line of the follower respectively.

65. Involute Cams.—Involute cams are so-called because the cam curve is usually of involute form. The follower is generally raised in less than half of the revolution of the cam and is brought back to the starting point by gravity or springs. Fig. 60 shows

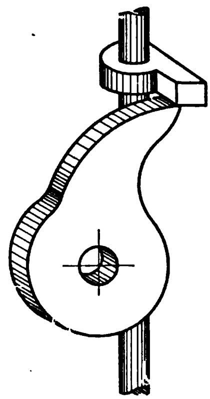


FIG. 60.

such a cam. One use for them is on ore crushers in stamp mills. Several cams are placed on the same shaft and so arranged that they will raise their respective rods at different times during the revolution of the shaft. The cam may be so designed that the follower is raised twice for each revolution of the shaft.

66. Method of Laying Out an Involute Cam.—An involute is the curve generated by a point in a taut string as it is unwrapped from a cylinder, or by a point on a straight edge that is rolled on a cylinder.

The circle from which the involute is developed is called the *base circle.*

Divide the angle ϕ, Fig. 61, through which the involute is to be developed into any number of preferably equal parts, as six and tangent to the base circle through these points draw the lines $1a$, $2b$, $3c$ . . . $6f$. Where the lengths of divisions on the base circle are not too great[1] the chord can be taken as

[1] If the lengths of divisions are taken equal to one-tenth the diameter of the base circle, they will be accurate to about one-thousandth of an inch per step.

the length of the arc. Lay off the chord 01 on 1*a* getting the point *a*. Twice the chord 01 on 2*b*, getting the point *b*, etc., then draw a smooth curve through the points *a*, *b*, *c*, . . . etc., which will be the involute curve.

The equation for the involute is $\phi = \dfrac{\text{Length of Arc}}{\text{Radius of Base Circle}}$, the angle ϕ being in circular measure.

In applying this to a cam follower, the amount that the follower is raised is equal to the length of the arc.

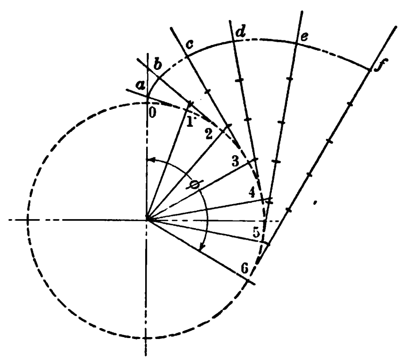


Fig. 61.

Problem.—Through how many degrees must an involute cam turn in order to raise its follower 3″ if the diameter of the base circle from which the involute is developed is 5″?

$$\phi \text{ (in degrees)} = \frac{3 \times 180}{2\frac{1}{2} \times \pi} = \frac{540}{7.85} = 68.79°.$$

The distance between the center of the cam and the center line of the follower should be equal to the radius of the base circle, so that contact between the cam and follower will be on the center line of the follower. The force exerted on the follower will then be more nearly all used in raising it.

Cams with Oscillating Followers

67. Cams with Oscillating Followers.—An oscillating follower is one in which points in it move in arcs of circles instead of in straight lines. Fig. 62 illustrates a roll follower cam of this type, while Fig. 63 shows a flat face or tangent follower cam.

The method of laying out a cam of this kind is somewhat more difficult than that for the reciprocating follower.

Let the angle ABC, Fig. 64, be the angle through which an oscillating follower is to be raised with a harmonic motion, while the cam turns through an angle of 120°; the follower to remain at rest for 150° of revolution, and to drop with harmonic motion in the remaining angle. In this figure OA is the radius of the

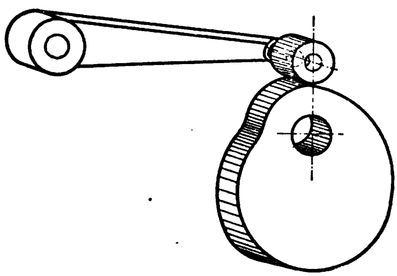


FIG. 62.

base circle, AB the radius of the follower arm, and the arc AC the path of the center line of the roll as the follower is raised.

As the center of the roll moves along AC with a harmonic motion which cannot be laid out directly on the arc, it is necessary to rectify the arc on a straight line and lay out the harmonic motion on that. Lay off on AD a length $A6$ equal to the arc AC. With $A6$ as a diameter lay out a semi-circle and divide it

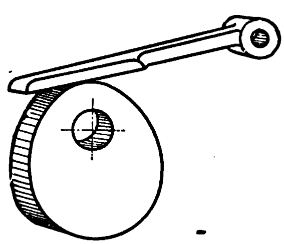


FIG. 63.

into any convenient number of equal parts as six, then project these points on the diameter, getting the points 1, 2, 3, 4, 5 and 6. Lay off these points on the arc, so that $A1' = A1$, $1'2' = 12$, $2'3' = 23$, etc. Through the points $1'$, $2'$, $3'$ . . . C and the center of the follower B, draw lines intersecting the line OD at 1_1[1], 2_1, 3_1, 4_1, 5_1 and 6_1.

[1] 1_1 and 2_1 are not shown on the figure on account of the smallness of the drawing, but they will be at the intersection of the lines through $1'B$, $2'B$ and OD.

Divide the first 120° of the cams circumference into six equal parts to correspond with the parts into which the semi-circle was divided.

With O as a center lay off $O1'' = O1_1$, $O2'' = O2_1$, $O3'' = O3_1$ . . . $O6'' = O6_1$. If the roll had moved along the line AD, instead of along the arc, the points $1''$, $2''$, $3''$ . . . $6''$ would be points on the theoretical cam curve, but since it moved on

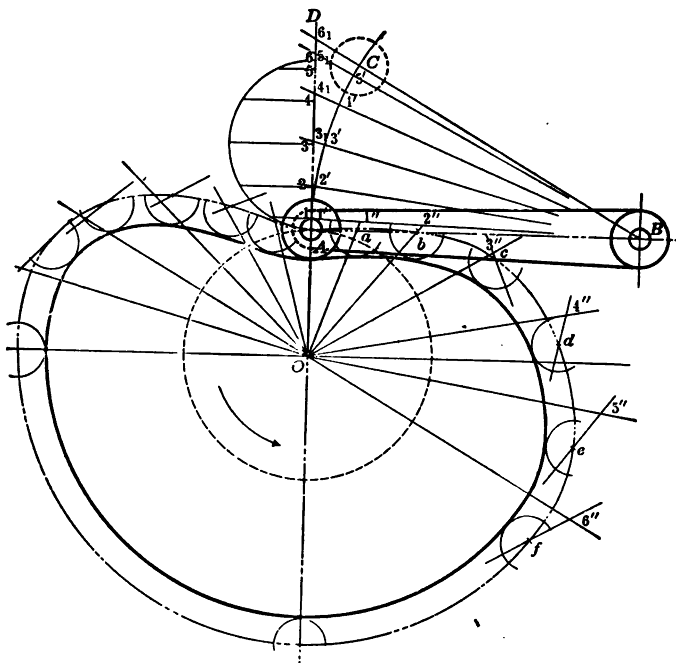


Fig. 64.

the arc and the lines $O1''$, $O2''$, $O3''$ . . . $O6''$ represent different positions of the line OD, it is necessary to make a correction for the distance that the center of the roll is away from the straight line.

This can be done by drawing through the points $1''$, $2''$, $3''$ . . . $6''$, lines making the same angles with $O1''$, $O2''$, $O3''$ . . . $O6''$ that 1_1B, 2_1B, 3_1B . . . 6_1B make with OD, and laying off $1''a = 1_11'$, $2''b = 2_12'$, $3''c = 3_13'$ . . . $6''f = 6_1C$.[1]

[1] The reason for making the correction can be seen if the student will take a piece of tracing paper or cloth and placing it over the figure, mark the center O. Then trace any line as $O5''$ and $5''e$. Revolve $O5''$ around O until it coincides with OD, when the point e will fall on $5'$.

A smooth curve drawn through the points *a*, *b*, *c*, *d*, *e* and *f* will give the theoretical cam curve. Taking points in this line and a radius equal to the radius of the roll the working curve can be found. The remaining part of the cam curve is found in a similar manner.

If a flat face follower is to be used the method is the same up to the point of finding the working curve, except that it will be necessary to draw the lines $1''a$, $2''b$, $3''c$ . . . etc., much longer so that they will form triangles as was done in Fig. 59.

68. There are a number of other methods of laying out this cam which will give the same results; one of these is shown in Fig. 65.

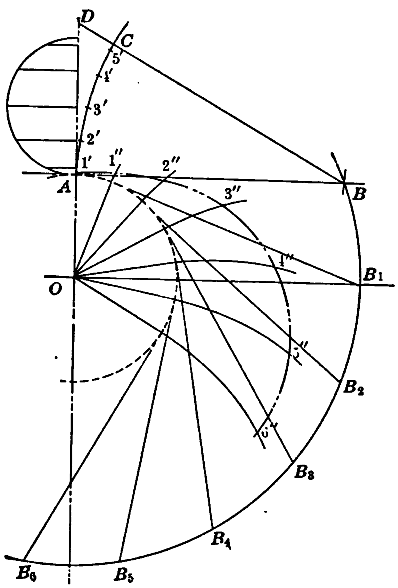


FIG. 65.

This method requires a much larger sheet of paper for the layout, but is less laborious if the facilities are at hand. Assume the data as in Fig. 64 and after locating the center of the oscillating arm *B* take a radius *OB*, and with *O* as a center, strike an arc. Divide the angle through which the cam is to turn to give the desired rise to the follower up into six equal parts, and through these points draw tangents to the base circle, intersecting the arc of radius *OB* at B_1, B_2, B_3 . . . B_6. With these points as centers and a radius *AB*, draw arcs from the base circle outward.

The points $1'$, $2'$, $3'$, $4'$, and $5'$ on the arc AC are found as in Fig. 64; with O as a center and radii $O1'$, $O2'$, . . . OC strike arcs, getting the points $1''$, $2''$, $3''$ . . . $6''$, which will be points on the theoretical curve.

PROBLEMS

24. Lay out the theoretical and working curves for a disk-edge cam that will raise a reciprocating roll follower 4 in. with harmonic motion in 180° of the cam's revolution; the follower to remain at rest for 90°, and to drop with uniform motion to the starting point in the remaining angle. Cam to revolve clockwise.

25. Lay out the theoretical and working curves for a disk-edge cam that will raise a reciprocating roll follower 3 in. with uniform motion in 120° of revolution; the follower to remain at rest for 60°, and to drop 3 in. with harmonic motion in the remaining angle. Cam to revolve counter-clockwise. Center line of follower 1 in. to left of cam shaft center.

26. Show the theoretical and working curves for a disk-edge cam that will raise a reciprocating flat-face follower 5 in. with uniform motion in 120° of the cam's revolution; the follower to remain at rest during the next 60°, and to drop with harmonic motion to the starting point in the next 180°. Cam to revolve counter-clockwise. Flat face of follower to make an angle of 75° with center line of follower.

27. (*a*) Design a disk-edge cam to revolve in a clockwise direction and give its $\left\{\begin{matrix}\text{roll}\\ \text{flat-face}\end{matrix}\right\}$ follower a reciprocating motion, uniformly accelerated upward during the first portion of the cam's revolution; a uniformly retarded motion upward during the second portion, and a uniform drop during the remaining portion.

(*b*) After laying out the curve as in (*a*), lay it out for the first two portions of the revolution according to the law of harmonic motion, the third portion being the same as before.

Data	1	2	3	4
Diameter of base circle, inches......	$4\frac{1}{4}$	$3\frac{3}{4}$	$4\frac{1}{8}$	$3\frac{5}{8}$
Diameter of roll, inches............	$1\frac{1}{4}$	$1\frac{1}{16}$	$1\frac{5}{16}$	$1\frac{1}{8}$
Diameter of roll pin, inches........	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{16}$
Width of follower, inches...........	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{11}{16}$
Rise of follower, inches............	$3\frac{3}{4}$	4	$3\frac{7}{8}$	$4\frac{1}{8}$
Diameter of cam shaft, inches......	$1\frac{1}{4}$	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{1}{8}$
Distance of center line of follower to left of cam-shaft center, inches.	1	0	$\frac{7}{8}$	$\frac{1}{2}$
First portion, degrees..............	90	90	60	120
Second portion, degrees............	90	90	60	120
Third portion, degrees.............	180	180	240	120

Note.—Make an ink drawing, full size, with center of cam shaft 7½ in. from top border line and in the center of the sheet right and left.

Use 32 divisions for Nos. 1 and 2 and 36 divisions for Nos. 3 and 4, laying them out upon a 12 in. diameter circle to insure greater accuracy.

Do not ink in this circle, and ink radial lines only about ½ in. farther out than roll centers.

If a flat-face follower is specified, disregard the dimensions relating to cam roll and pin, and make the length of follower face ¼ in. longer than length of contact on each side of center line.

The object of drawing the harmonic curve superimposed on the uniformly accelerated and uniformly retarded curve is to see the difference between them. Time, 6 hours.

28. An involute cam follower is to be raised 5½ in. Distance between center of cam and center line of follower is 6 in. Through how many degrees must the cam turn?

29. What should be the distance between the center of an involute cam and the center line of its follower if the follower is raised 7 in. in 100 degrees of revolution?

Positive Motion Cams

69. Positive Motion Cams.—A positive motion cam is one that does not depend upon the force of gravity, springs or any other external means to bring the follower back to its initial position.

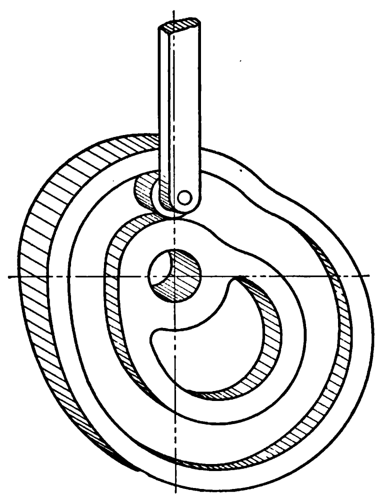


Fig. 66.

A common method of making a positive motion cam where a roll follower is used, is to have the roll move in a groove in the side of a disk or plate as is shown in Fig. 66.

70. Cams of Constant Diameter.—A cam of constant diameter is one in which the distance between the sides, measured through the center of the cam shaft, is constant. It has the disadvantage that any desired law of motion can be laid out for but 180°, the second 180° being laid out to correspond. The layout of a cam of this type is shown in Fig. 67. Lay out the first 180° of the

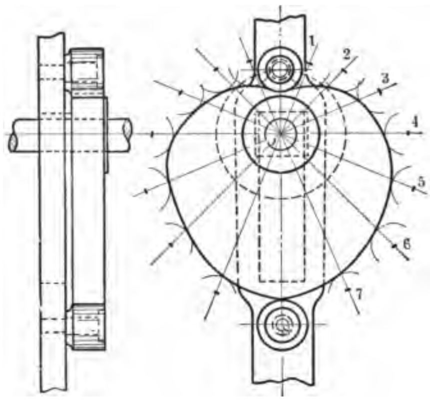


FIG. 67.

cam for the desired law of motion, getting the points 1, 2, 3 . . . 8 on the theoretical cam curve. The distance from the starting point through the center of the cam shaft to the point 8 (not shown in Fig.), will be the distance between the centers of the rolls. With this length as a radius and centers at 1, 2, 3 . . . 7 strike arcs intersecting their respective radii drawn through the center of the cam. These intersections will give points on the theoretical curve for the second half of the cam.

This type of cam can be used for heavier work than the groove cam already mentioned.

71. Main and Return Cam.—As the name implies, this consists of two cams. The main cam is laid out for any desired law of motion for a complete revolution or 360°. Lay out the points 1, 2, 3 . . . 16 on the theoretical curve, Fig. 68, for any desired law of motion and choose a distance that the rolls are to be apart. With this distance as a radius and centers at 1, 2, 3 . . . 16 strike arcs on the radii diametrically opposite the points, which will give points on the theoretical curve of the second cam. The working curves may then be found after choosing rolls of any convenient diameter.

In this case as in that of the constant diameter cam, the follower is generally constructed so that it envelops the shaft and is guided by a square bushing. This cam combination is particularly desirable for heavy work and slow speeds.

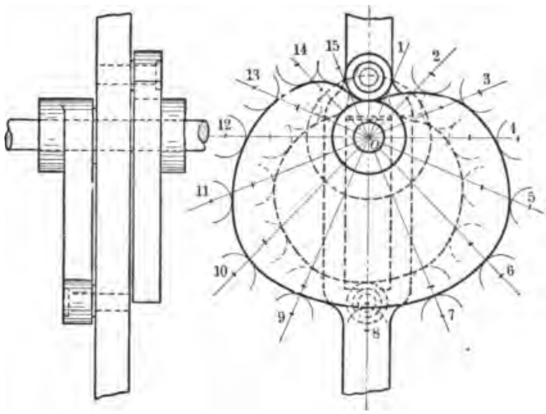


FIG. 68.

72. Cams of Constant Breadth.—A constant breadth cam is one in which the distance between parallel faces is constant. It is used with a flat face follower, and as in the case of the constant diameter cam, can be laid out for any desired law of motion,

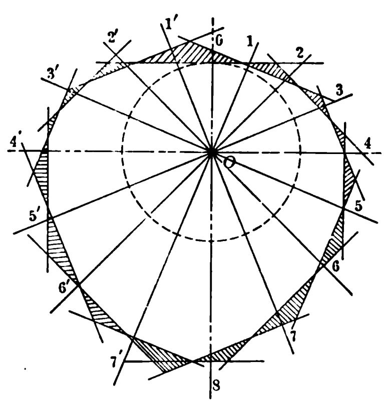


FIG. 69.

for 180° only. Assume that the points 1, 2, 3 . . . 8, Fig. 69, are points on the theoretical cam curve. The distance 08 determines the distance between the parallel sides of the follower

faces. Through the points 1, 2, 3 . . . 8 draw lines perpendicular to the radial lines, and on the opposite side of the cam center locate the points 7′, 6′, 5′ . . . 1′ by using 1, 2, 3 . . . 7 as centers and a radius 08. Through the points 1′, 2′, 3′ . . . 7′ draw lines parallel to those drawn through 1, 2, 3, . . . 8. To obtain the working curve draw a smooth curve tangent to the middle points of the triangles formed by these perpendicular lines, as was done in Fig. 59.

It will be noted that the above construction is similar to that of the constant diameter cam up to the point of finding the theoretical cam curve.

Fig. 70 is a special type of constant breadth cam used on light mechanisms such as sewing machines.

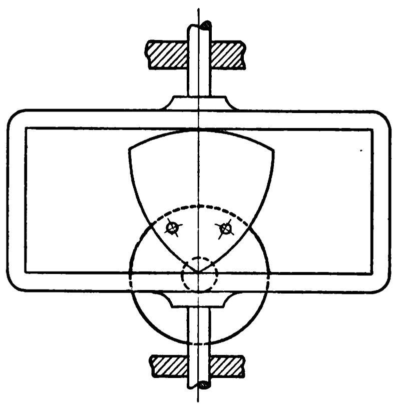


FIG. 70.

Lines joining the three points of the cam form an equilateral triangle. The sides are arcs of circles with centers at the points of the triangle, and radii equal to the chordal distances between the points.

73. Cylindrical Cams.—These cams may be used for either rectilinear or oscillating motion.

The motion is obtained by a groove in the cam as in Fig. 71 or by fastening strips on the surface of the cylinder as in Fig. 72. The latter case is used on automatic machines such as screw machines for operating the turrets and wire feeds, where it is desired to do different kinds of work. The same cam can be

used by making a different adjustment of the strips, instead of using a new cam complete.

74. Cylindrical cams can be laid out in several different ways, one of which is to lay out the cam without developing the cylindrical surface, and another is to develop the surface before laying out the curve.

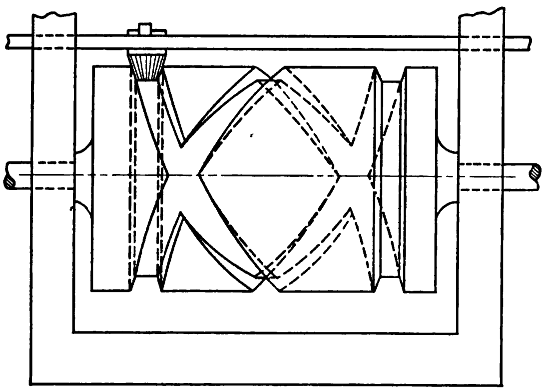


FIG. 71.

A problem will illustrate these two methods.

Problem.—Lay out the theoretical curve for a cylindrical cam that will move a reciprocating follower a distance of 12 in. to the right with a uniform motion in 1¼ turns of the cam; the follower to remain at rest for ¼ turn; to move 12 in. to the left

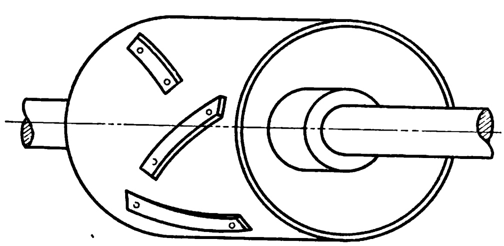


FIG. 72.

with a uniformly accelerated and uniformly retarded motion in one turn, and to remain at rest for ½ turn, thus bringing the follower back to the starting point.

Lay out the end and side views of the cylinder, Fig. 73, and divide the circle representing the end of the cylinder into any

convenient even number of equal parts, as eight. Divide the 12 in. on the side view into the same number of equal parts as there are equal divisions in $1\frac{1}{4}$ revolutions on the end view, or ten, and number all of the divisions to correspond. Through the points on the side view draw lines across the cylinder, perpendicular to its axis. Then through the points 1, 2, 3 . . . 8 on the end view, draw lines parallel to the axis of the cylinder until they intersect the lines drawn through the points 1, 2, 3 . . . 8 on the side view, thus getting the points 1′, 2′, 3′, etc.

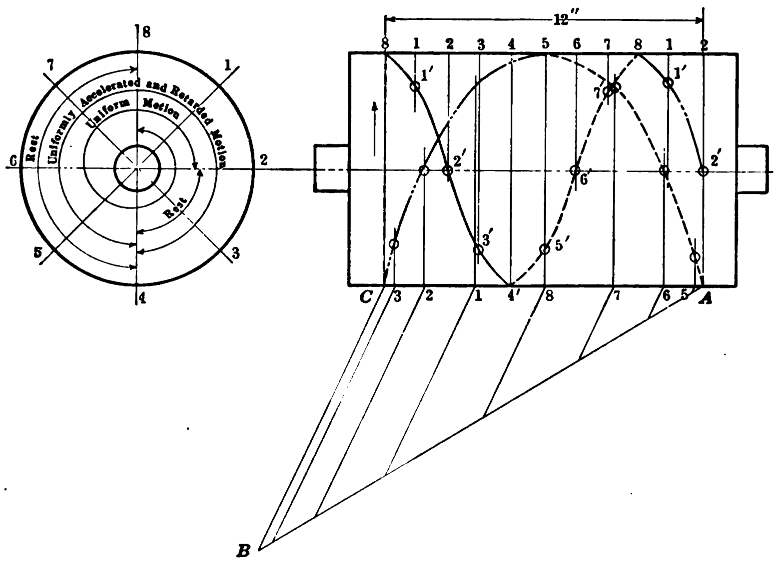


FIG. 73.

Assume that the top of the cylinder moves away from you as you look at the side elevation; then the curve will pass across the front of the cylinder and disappear at 4′, and will be on the far side until the point 8 is reached, when it will again pass in front.

From 2′ to A the follower is to remain at rest, so the curve can have no motion along the cylinder.

During the next revolution the follower is to move 12 in. to the left with uniformly accelerated and uniformly retarded motion. From A draw a line AB of indefinite length and lay off on this line eight divisions in the series 1–3–5–7–7–5–3–1, using any convenient unit.

Through *B* and the point *C*, 12 in. away from *A* draw a line. Parallel to *BC* and through the points on *AB* draw lines intersecting *AC*. Through these intersections on *AC* and perpendicular to the axis of the cylinder draw lines, and project across from the points on the end view to get points on the theoretical curve as before.

From this point the follower is to remain at rest for ½ revolution of the cylinder, so that the theoretical curve will be drawn from *C* to 8, perpendicular to the axis.

75. In order to lay out this same cam on a developed surface, lay out the circumference and length of the cylinder, as in Fig. 74, and divide the circumference into the same number of

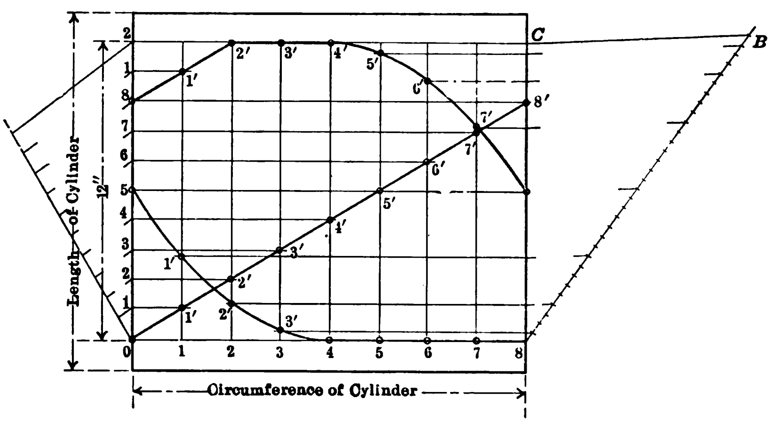


FIG. 74.

equal parts as there are divisions in the end view of Fig. 73, or eight.

Divide the length of the cylinder up, according to the law of motion desired, and obtain points on the theoretical curve as shown on the figure.

In cams of this kind where the groove crosses itself, the part that follows in the groove, which in disk-groove cams is a roll, but in cylindrical cams is the frustrum of a cone, must be of special shape.

It is generally made oblong and of such a length that the leading end will be across the intersecting groove and well entered in the original groove before the widest part of the follower has reached the intersecting groove.

In order to obtain best results with a cylindrical cam the angle that the theoretical curve on the developed cylinder makes with a perpendicular to the axis of the cam should not be more than 30°. If the angle exceeds this, make the diameter of the cylinder larger.

76. Cylindrical Cam with Oscillating Follower.—The layout for a cam of this type is shown in Fig. 75. The radius of the follower is *OA* and turns through the angle *AOB*. The other data is the same as in the previous case. It will be seen that the elements into which the cylinder is divided, instead of being parallel to the axis as in the case of the reciprocating follower,

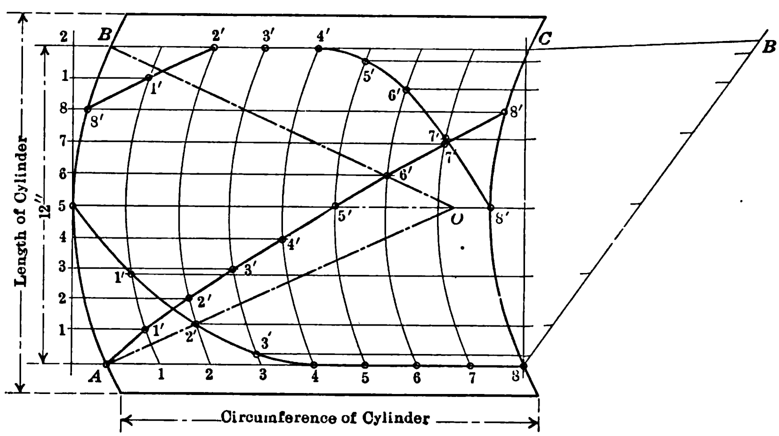


FIG. 75.

are arcs of circles, of radii *OA* and with centers on a line through *O* perpendicular to the axis of the cylinder. Other than this, there is no difference between the methods of laying out the oscillating and reciprocating follower cams.

77. Inverse Cams.—An inverse cam is one in which the roll is on the driver and the groove in the follower. Fig. 76 shows a cam of this type.

The inversion is made to avoid "dead points" that would sometimes occur when using an ordinary cam combination. It has the disadvantage that the motion can be laid out for only 180 degrees. It is largely used for light mechanisms such as sewing machines.

Fig. 77 represents a "Scotch cross-head" which is a special case of an inverse cam with a reciprocating follower, and often used on fire engines. In this case the groove is straight and perpendicular to the center line of the follower.

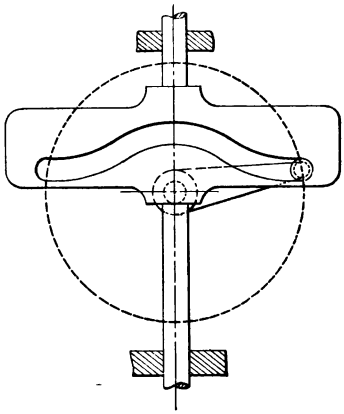


Fig. 76.

The method of laying out the inverse cam can be illustrated by a simple problem.

Problem.—Lay out the theoretical and working curves for an

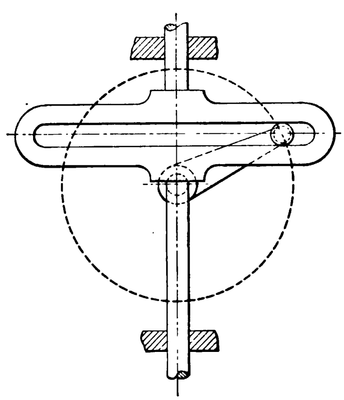


Fig. 77.

inverse cam that will give its follower motions as follows: 1 in. raise during the first 20° of revolution of driving arm; ½ in. raise during the next 40°; rest during the next 30°; ¾ in. drop during the next 30°; 2 in. raise during the next 40°, and rest during the

next 20°. Driving arm to turn counter clockwise. Length of driving arm 6 in.

Draw a circle, Fig. 78, of radius $OA = 6$ in. to represent the path of the center of the roll. Draw $O1$, $O2$, $O3$, $O4$, $O5$ and $O6$ to represent the different positions of the center line of the driving arm, and through 1, 2, 3, 4, 5 draw lines perpendicular to the horizontal diameter of the circle of radius OA, thus getting the points $1'$, $2'$, $3'$, $4'$, $5'$. If the groove were horizontal and straight as in Fig. 76, when the driving arm had passed through the angle $AO1$ with its initial position, the follower would have been

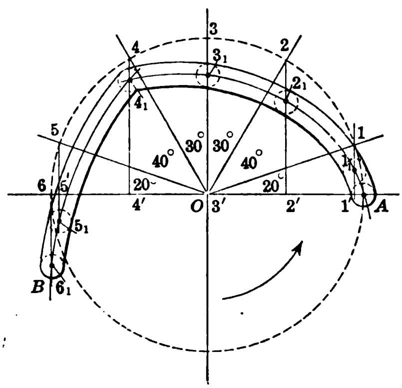


FIG. 78.

raised a distance $11'$, but since the follower is to be raised but 1 in. in that angle, the groove must be curved upward the difference between $11'$ and 1 inch, or the distance $1'1_1$. The point 1_1 can be found directly by laying off on the line $11'$, from 1, 1 in.

When the driving arm passes through the angle $1O2$, the follower is to be raised ½ in., and since it has already been raised 1 inch, lay off 1½ in. on $22'$, getting the point 2_1. During the next angle the follower remains at rest, so 3_1 will be 1½ in. down from 3.

While the follower is passing through the angle $3O4$, the follower drops ¾ in., and as we already had a raise of 1½ in., the net raise laid off from 4 will be ¾ in.

A 2 in. raise during the next 40° makes 2¾ in. to be laid off from 5. During the last 20° the follower remains at rest, so 2¾ in. will also be laid off from 6, to get the point 6_1.

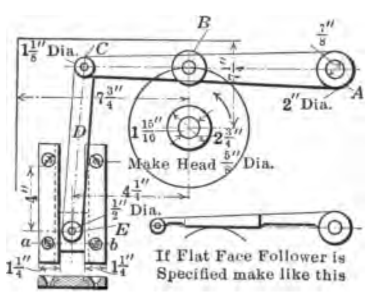
The points 5_1 and 6_1 in this problem fall below the original position of the groove at A, but since the follower is $2\frac{3}{4}$ in. higher than it was there, the groove must be the same amount lower for the driving arm is horizontal in both cases. Draw a smooth curve through the points 1_1, 2_1, 3_1 . . . 6_1 to obtain the theoretical curve. From 2_1 to 3_1 and from 5_1 to 6_1 the follower remains at rest so these parts of the curve should be arcs of circles of radii OA, and a center on the center line of the follower.

The working curve is found by choosing a diameter of roll and with centers on the theoretical curve draw circles the diameter of the roll, then draw curves tangent to them.

During the last 180° of revolution of the driving arm, the roll passes through the groove in the opposite direction, and causes the follower to go very much below its original position. If this is not desirable, the ends of the groove can be left open and the roll leave the groove during the last 180°.

PROBLEMS

30. **Design an oscillating arm cam to raise the sliding block 4 in. with harmonic motion in 180° of counter-clockwise revolution of the cam; the block to remain at rest for 45°, and to drop 4 in. with a uniform motion in the remaining angle.**



Section at *a-b*

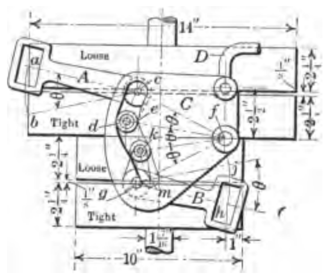
Data: $AB = 9\frac{1}{2}''$; $BC = 4\frac{1}{2}''$; $CE = 8\frac{5}{8}''$
Diameter of base circle $= 6''$
Diameter of cam roll $= 1\frac{7}{16}''$
Diameter of cam roll pin $= \frac{1}{2}''$

***Note.*—Make a full size ink drawing and leave off all dimensions. Divide the 180° angle into 12 equal parts and the 135° angle into 10 equal parts. Time, 6 hours.**

31. **Make a full-size ink drawing of the belt-shifter cam. The shifter arms to move with uniform acceleration and retardation about their centers**

c and g while the cam turns about its center f. Diameter of cam rolls 1 in. Diameter of shifter arm studs $\frac{5}{8}$ in. θ = angle through which shifter arms move.

Note.—Find the centers c and g on a line parallel to the pulley shaft and midway between the outer ends of the shifter arms. The angle $cfg = 30$, and f is located by drawing through c, downward to the right, a line making an angle of $1\frac{1}{2}\theta$ with the horizontal, and a line through g upward to the right and also making an angle of $1\frac{1}{2}\theta$ with the horizontal. The point f is at the intersection of these lines.



Divide cfg into three equal parts. With f as a center and a radius fc strike the arc cd, thus locating d. Lay off $dce = \theta$. With c as a center and a radius cd strike the arc de. Then d and e are the two extreme positions of the cam roll. The movement of the other roll is the same. Divide the angle efk into six equal parts and the arc de into corresponding parts for uniformly accelerated and uniformly retarded motion. Time, 6 hours.

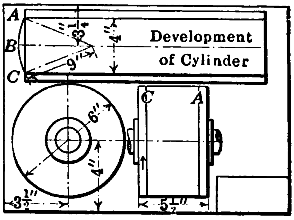
32. Lay out the theoretical and working curves for a main and return cam that will give a reciprocating follower a harmonic rise of 6 in. in 90° of revolution, to remain at rest for 180°, and a uniform drop during the remaining angle. Cam shaft revolves clockwise.

33. Lay out the theoretical and working curves for a constant-diameter cam that will raise a follower 8 in. with harmonic motion in 60°; rest for 60°, and a uniform rise of 3 in. in the next 60°. Cam revolves clockwise. If the diameter of the base circle is 6 in. what is the correct distance between the rolls?

34. Make a sketch showing the theoretical curve for a cylindrical cam that will give a reciprocating follower a harmonic motion of 8 in. to the left in one revolution of the cam; to remain at rest for $\frac{1}{2}$ revolution; to move uniformly to the left 2 in. in $\frac{1}{2}$ revolution, and a uniform motion back to the starting-point in the least number of revolutions. Top of cam to turn away from you as you look at side elevation.

35. Lay out the theoretical and working curves for a cylindrical cam that will give its oscillating follower a harmonic motion from A to C in $\frac{1}{3}$ revolution; the follower to remain at rest for $\frac{1}{6}$ revolution; to return from C to A in $\frac{1}{3}$ revolution with uniform motion, and remain at rest for $\frac{1}{6}$ revolution.

Diameter of base of roll $\frac{3}{4}$ in.
Depth of groove $\frac{1}{2}$ in.



Note.—Make the views as shown, in ink and full size. Put no statement of the problem on the sheet. Time, 8 hours.

36. Make a sketch showing the theoretical curve for an inverse cam that will raise its reciprocating follower 1 in. during the first 30° revolution of the driving arm; to raise $\frac{1}{2}$ in. during the next 30°; to rest during the next 30°; to raise 2 in. during the next 20°; to drop $1\frac{1}{4}$ in. during the next 20° and to raise $1\frac{1}{2}$ in. during the next 50°. Driving arm $6\frac{1}{2}$ in. long and turns counter-clockwise.

CHAPTER VII

GEARING FOR PARALLEL SHAFTS

78. There are three separate cases for which gears can be used.

First. To connect parallel shafts. { *a.* Constant velocity ratio. *b.* Variable velocity ratio.

Second. To connect intersecting shafts. { *a.* Constant velocity ratio. *b.* Variable velocity ratio.

Third. To connect shafts that are neither parallel nor intersecting.

The first two cases with a constant velocity ratio are the most common.

Friction Gearing

79. Friction gearing may be said to be gearing in which the motion or power that is transmitted depends upon the friction between the surfaces in contact. In this case the power that can be transmitted at a constant velocity ratio is limited, for as soon as slipping occurs, the velocity ratio changes.

80. Rolling Cylinders.—This is the most simple friction gear combination and is used to transmit motion between parallel shafts with a constant velocity ratio. Fig. 79 shows such a combination.

Velocity Ratio.—It has been stated before (Art. 15) that the angular velocities of two points in two bodies having the same linear velocities, but different radial distances from their centers of rotation, are inversely as their radii. Thus if the two cylinders of Fig. 79 rolling together without slipping, their revolutions are inversely as their radii. In order to have a constant velocity ratio, there must be pure rolling contact between

the cylinders. This means that the consecutive points or elements on the surface of one must come in contact with the successive points or elements on the surface of the other in their order, or in other words there must be no slipping.

81. Grooved Cylinders.—In order to transmit more power than can be obtained by smooth cylinders, they are sometimes grooved

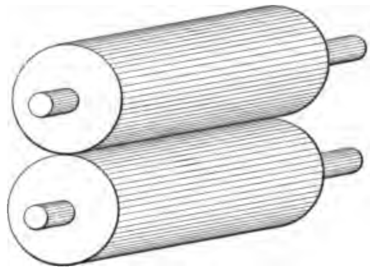


Fig. 79.

as shown in Fig. 80. With this arrangement there is not pure rolling contact between the cylinders, except at a point about midway along the depth of the grooves.

The angle included between the sides of the grooves should not be more than 30° or less than 10° in order to obtain best results. If the angle is made too great, the effect of the grooves

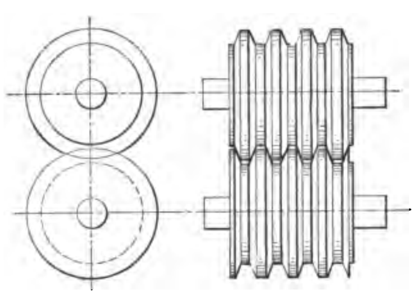


Fig. 80.

will be lost and the action will be more like the case of plain cylinders, while if the angle is made too small, there will be a tendency for the cylinders to wedge themselves together, and it will require excessive power to drive them.

82. Evan's Friction Cones.—This combination, shown in Fig. 81, is for connecting parallel shafts to obtain a variable velocity ratio. On the shafts that are to be connected are placed the

frustrums of two similar cones, A and B, with the large end of one opposite the small end of the other, and between them an endless belt C, which is held in place by a belt shipper, or other similar device.

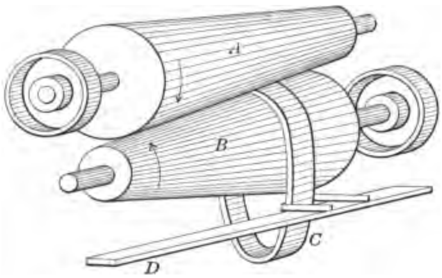


FIG. 81.

Velocity Ratio.—If A is the driver, the revolutions of B will be inversely as the radii of the cones where they are in contact with the belt. Thus if the radius of the large ends of the cones is 4 in. and that of the small ends 2 in., when the belt is at the large end of the driver, the revolutions of the driven will be twice those of the driver, while if the belt is at the small end of the driver and the large end of the driven, the revolutions of the driven will be one-half those of the driver.

It can also be seen from the figure that the direction of rotation, or the *directional relation* is opposite in the two shafts. If it is desired that they turn in the same direction, a small wheel can be substituted for the belt. This wheel should be covered with some soft material, such as rubber or leather, to make good frictional contact. This combination is used on machine tools, such as grinders and speed lathes, for obtaining variable speeds.

83. Seller's Feed Disks.—This is also a type of friction gear for connecting parallel shafts to obtain a variable velocity ratio and is shown in Fig. 82. The disks A and C are on the shafts to which it is required to give the desired velocity ratio.

Two disks B, having their inner surfaces slightly convex, fit loosely on the same shaft and engage the sides of A and C near their rims, between the convex surfaces.

On the shaft with the disks B, between the frame and the back of the disks, are helical springs which tend to push the disks together and exert pressure on the sides of A and C.

Velocity Ratio.—If A is the driver, the revolutions of C can be increased by moving the intermediate disks B, toward A.

This combination derives its name from the fact that it was first used on the feed mechanisms of machine tools built by the William Sellers Company of Philadelphia.

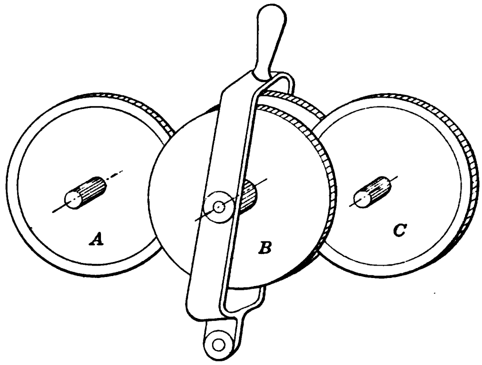


FIG. 82.

84. Bevel Cones.—These cones of which Fig. 83 is an illustration are used for transmitting motion between shafts that intersect. The cones need not be of the same size, and the shafts need not intersect at a right angle.

Velocity Ratio.—If the velocity ratio is as 1:1, the diameters of the bases of the cones must be the same diameter. In order to

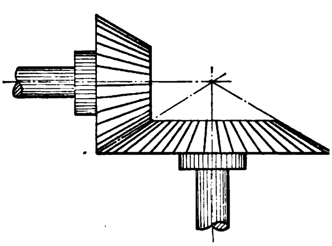


FIG. 83.

lay out a pair of cones for any velocity ratio and shaft angle, the method is as follows:

Assume that the driver is to make 5 revolutions while the driven makes 7 and that the shaft angle is 75°. Let OA, Fig. 84, be the axis of the driver and OB that of the driven. At any point as a on OA draw a perpendicular line ab and on this line lay off 7 units; also at any point as c on OB draw

the perpendicular line cd, and on it lay off 5 units of the same size as those laid off on ab. Through b and d draw lines parallel to OA and OB respectively; through their intersection at C, draw the line CO. This line will be an element of the two cones, and cones of any diameter may be drawn as indicated in the dotted outlines, by using CO as the common element.

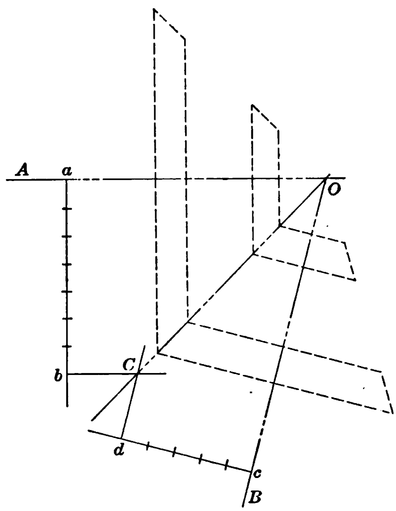


FIG. 84.

85. Brush Plate and Wheel.—This form of friction gearing shown in Fig. 85 is for the purpose of transmitting variable velocity ratio between shafts that intersect at right angles. It consists of a large disk A and a small wheel B, which can be moved across the face of A, by means of a feather key in the shaft of B. The small wheel is usually made of some softer material than the disk which is generally of iron, in order to secure more friction between the surfaces.

The small wheel should be the driver so that in case the load becomes excessive and there is slipping, the small wheel will be worn down an equal amount all around. If the large disk were the driver and the load became excessive, a flat spot would be worn on the small wheel, which would then have to be trued up.

Velocity Ratio.—The velocity ratio depends upon the diameter of the small wheel and the *effective* diameter of the disk, that is, the diameter of the path that the small wheel makes on the disk,

The brush plate is used on a number of forms of sensitive drills and is useful in other cases for light loads where slight variations in speed are desired.

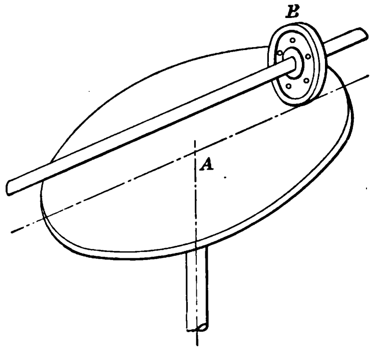


FIG. 85.

TOOTHED GEARING

86. In order to transmit more power than can be obtained with the rolling cylinders of Fig. 79 let us assume that projections are put on cylinder A, Fig. 86, parallel to the axis, and corresponding

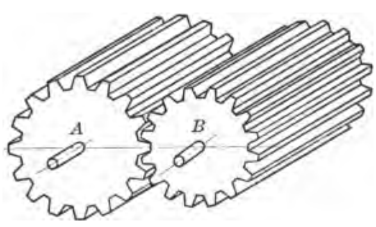


FIG. 86.

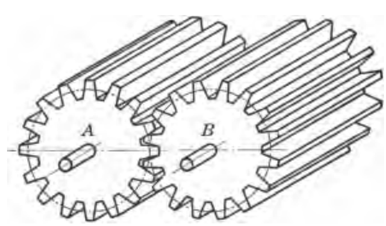


FIG. 87.

grooves in cylinder B. Between the grooves on B, put projections, and corresponding grooves on A. The cylinders will then appear as in Fig. 87.

The end view of the original cylinders will be imaginary circles, and are called the *pitch circles*, and their point of tangency is the *pitch point.*

Velocity Ratio.—The velocity ratio of a pair of gears will be inversely as the radii of the pitch point of the pitch circles. *In order that a pair of gears transmit a constant velocity ratio, their tooth curves must be such that a normal to the common tangent of the teeth at the point of contact will always pass through the pitch point.* This statement is called the fundamental law of gearing. The velocity ratio in this case will be constant for any fraction of a revolution, or, in other words, the pitch circles have pure rolling contact (Art. 80).

Two gears the centers of which are at A and B, Fig. 88, and pitch point P have a pair of teeth in contact at T as shown.

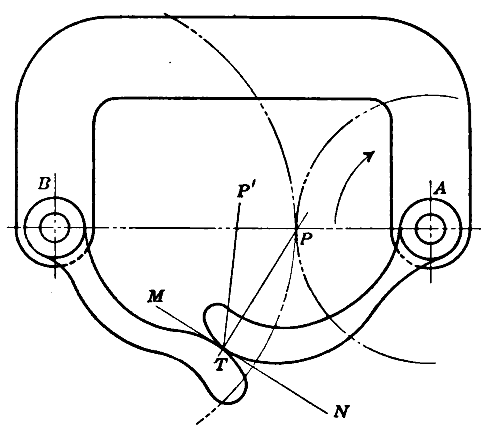


FIG. 88.

Their tooth curves are so formed that a normal to the common tangent MN through T also passes through the pitch point P.

The same relative motion between the tooth curves is obtained by holding one gear stationary and revolving the other gear and the frame, as by holding the frame stationary and revolving the gears. Suppose that the gear whose center is at B is held stationary and the one with a center at A revolved. Then the direction of motion of the contact point T of A is at right angles to PT, or along the line MN, and therefore in instantaneous contact with the contact portion of B.

Suppose that the curves are so formed that the common normal passes through any other point P' instead of P and let A be revolved as before in the direction indicated by the arrow: then A and B will break contact, because the contact point T of A moves toward M along the line MN as before, which is not the direction of the contact portion of B. If A is revolved in the opposite direction, the contact point T of A will move toward N, and the tooth curves will interfere, hence the fundamental law of gearing as stated above.

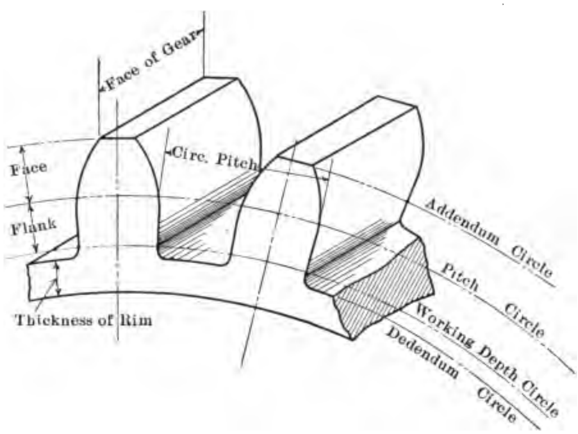


FIG. 89.

87. Definitions of Tooth Parts.—The *addendum circle* is the circle bounding the ends of the teeth. See Fig. 89.

The *working depth circle* is the circle *below* the pitch circle, a distance equal to that of the addendum circle *above* the pitch circle.

The *dedendum circle* or *root circle* is the circle bounding the bottoms of the teeth.

The *clearance* is the distance between the working depth and dedendum circles.

The *face* of the tooth is that part of the tooth lying between the pitch and addendum circles.

The *flank* of the tooth is that part of the tooth lying between the pitch and dedendum circles.

The *thickness of tooth* is its width measured on the pitch circle.

The *width of space* is the space between the teeth measured on the pitch circle.

Backlash is the difference between the thickness of a tooth and the space into which it meshes, measured on the pitch circles.

Pitch diameter is the diameter of the pitch circle, and is the diameter that is used in making all calculations for the size of gears.

Pitch is the measure of the size of the teeth. There are two kinds of pitch in general use for calculating gear teeth sizes, circular and diametral.

Circular pitch is the distance in inches between similar points of adjacent teeth measured along the pitch circle. It is the sum of the thickness of tooth and width of space, measured on the pitch line.

As there must be a whole number of teeth on the circumference of a gear, it is necessary that the circumference of the pitch circle divided by the circular pitch be a whole number. Thus it is seen that if the circular pitch is a whole number, the diameter of the pitch circle will usually be a number containing a fraction.

Let $P' =$ circular pitch in inches
Let $D =$ pitch diameter
Let $T =$ number of teeth

Then $TP' = \pi D$ or $T = \frac{\pi D}{P'}$; $P' = \frac{\pi D}{T}$; $D = \frac{TP'}{\pi}$. Thus if $P' = 1\frac{1}{2}$ in., $T = 36$, the pitch diameter $D = \frac{36 \times 1\frac{1}{2}}{3.1416} = 17.19$ in. very nearly.

Diametral pitch is the number of teeth on the gear per inch of diameter of the pitch circle.

Let $P =$ diametral pitch
Let $T =$ number of teeth
Let $D =$ pitch diameter

Then $T = PD$; $D = \frac{T}{P}$; $P = \frac{T}{D}$.

In a gear having 64 teeth and a pitch diameter of 16 in., the diametral pitch $P = \frac{64}{16} = 4$. Meaning that for every inch of diameter of the pitch circle, there are 4 teeth on the gear.

Diametral pitch is more generally used than circular pitch for the reason that the pitch diameter comes out in equal frac-

tions, and the pitch is deduced from the diameter and number of teeth without having to use the constant π, which is necessary when using circumferences.

It is sometimes desirable to convert from one pitch to the other. This can be done as follows:

In a given gear by the circular pitch system $T = \frac{\pi D}{P'}$, while if using the diametral pitch, $T = PD$.

So that we may write $\frac{\pi D}{P'} = PD$.

In the same gear by the two systems, the pitch diameter D will be the same, and as it appears in both sides of the equation may be canceled out.

Then $\frac{\pi}{P'} = P$ or $\pi = P'P$.

So that *circular pitch* $\times$ *diametral pitch* $= \pi$.

The *angle of action* is the angle through which a gear turns while a tooth on it is in contact with its mate on the other gear.

The *arc of action* is the arc subtending the angle of action.

The *angle of approach* is the angle through which a gear turns from the beginning of contact of a pair of teeth until contact reaches the pitch point.

The *angle of recess* is the angle through which a gear turns while contact in a pair of teeth is from the pitch point until the teeth pass out of contact.

Angle of Approach + *Angle of Recess* = *Angle of Action*

There are two systems of generating tooth curves in general use that will give a constant velocity ratio—the *involute* system and the *cycloidal* system.

PROBLEMS

37. In a brush plate and wheel combination, the diameter of the small wheel which is the driver is 4 in., and makes 60 r.p.m. The minimum effective radius of the brush plate is $1\frac{1}{2}$ in., and its maximum effective radius is 15 in. What are its revolutions in each case provided there is no slipping?

38. Let P = diametral pitch, P' = circular pitch, T = number of teeth and D = pitch diameter.

1. Given $P = 10$; $D = 5$. Find T.
2. Given $D = 15$; $P' = 2$. Find T.
3. Given $T = 48$; $D = 4\frac{1}{2}$. Find P'.
4. Given $P' = 1\frac{1}{4}$; $T = 30$. Find P.

39. In two spur gears the driver turns five times while driven turns three

times. Circular pitch $= \frac{7}{8}$ in. Number of teeth in driven 30. Find distance between centers and number of teeth in driver.

40. Two spur gears are in mesh. Teeth in driver, 80. Circular pitch $= 1\frac{1}{2}$ in. Distance between centers of shafts $= 20$ in. Find pitch diameter of, and number of teeth in, driven.

41. The distance between the centers of two spur gears that are in mesh is 20 in. If one gear makes three times as many revolutions per minute as the other how many teeth in each if the diametral pitch is 4?

42. In an annular gear and pinion combination, the distance between their centers is 8 in. Teeth in pinion $= 36$. Diametral pitch $= 4$. What are the diameters of, and how many teeth in each of the gears?

43. Two spur gears in mesh have 80 and 50 teeth respectively and are of $1\frac{1}{2}$ in. pitch. What is the proper distance between their centers?

Involute System

88. In the involute system the tooth outlines are involute curves.

The construction for the involute was shown in Fig. 61. As far as the generation of the involute curve is concerned, it makes no difference whether the band or string is unwrapped

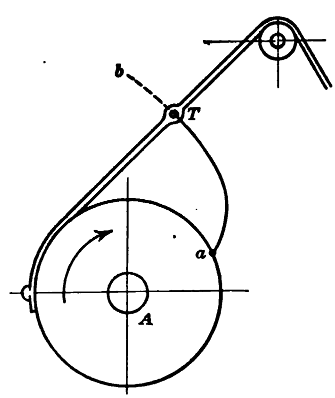


Fig. 90.

from a stationary cylinder, as was done in that case, or the band pulled in a straight line and the cylinder allowed to revolve as in Fig. 90. In this figure the cylinder A revolves about its center, and the tracing point T in the band traces the involute curve ab.

Now suppose that a second cylinder B, is used Fig. 91, and the band unwrapped from A to B. As it is unwrapped from

A, the tracing point T will trace the involute aT and at the same time it will trace the involute dT as it is wrapped on B. The dotted portions of the curves Tb and Tc are the parts of the involutes that will be traced as the cylinders are revolved further.

89. It is a property of the involute that a normal to the curve is tangent to the base circle. The two curves have a common normal at the point of contact T, which is tangent to both base circles. This line which is the locus of all the points of contact is called the *line of action.*

90. Spur Gears.—Let A and B, Fig. 92, be the centers of two spur gears with pitch radii AP and BP respectively.

The radii of the base circles from which the involutes are developed are obtained by drawing the line of action ab and dropping

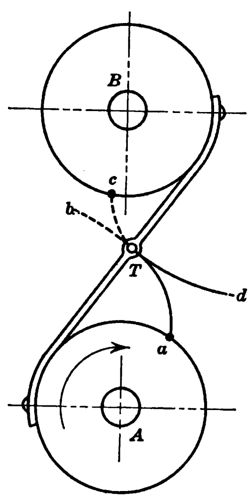


FIG. 91.

perpendiculars to it from the centers of the gears. The lengths of these perpendiculars are the radii of the base circles.

The *angle of obliquity* α, is the angle that the line of action makes with a tangent to the pitch circles drawn through the pitch point.

If the angle of obliquity is zero the base circles will coincide with the pitch circles, and the teeth of the gears will act only at the pitch point. The angle of obliquity can be taken any angle more than zero, but in practice it is usually taken such

that its sine is 0.25 which corresponds to angle of approximately $14\frac{1}{2}°$.[1]

91. In the figure, let the gear with its center at A be the driver and revolve clockwise as shown. There should be no contact between the teeth of A and B until the point a, where the line of action is tangent to the base circle of the driver, is reached, as this is the point where the involute is first begun to be generated. The limit of contact on the other side of the pitch point

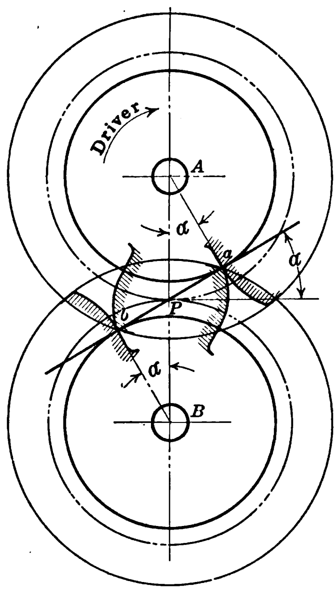


FIG. 92.

is b, where the line of action is tangent to the base circle of the driven gear. Contact will be along the line ab, beginning at a and ceasing at b provided the teeth of the driven are long enough to reach the point a, and the teeth of the driver of sufficient length to reach the point b.

If the teeth of the gears are not long enough for contact to begin at a and cease at b, it will begin where the addendum

[1] Several years ago there was brought out a system of involute gear teeth known as the "stub tooth" gear in which the angle of obliquity is 20°. These teeth are wider and shorter than those of the standard involute tooth. For a discussion of this form of tooth see a bulletin by the Fellows Gear Planer Company on "The Stub-Tooth Gear."

circle of the driven cuts the line of action and cease where the addendum circle of the driver cuts the line of action. Contact between a pair of teeth should be continuous between the beginning and end of contact along the line ab.

Contact between the teeth of any pair of gears that are in mesh always *begins* between the *driver's flank* and the *follower's face* and *ceases* between the *driver's face* and the *follower's flank.*

For a pair of gears having equal arcs of approach and recess to have continuous contact, that is, not to have one pair of teeth cease contact before a second pair have begun contact the angle $PAa = \alpha$, Fig. 92, should not be less than $\frac{180°}{T}$ where T equals the number of teeth in the smaller gear.

It is desirable to have more than one pair of teeth in contact at one time, or to *have the pitch arc less than the arc of action.* Thus if the pitch arc is one-half the arc of action, two pairs of teeth will be in contact at one time.

The maximum pitch angle equals $2PAa = 2\alpha = \frac{360°}{T}$ or $\alpha = \frac{180°}{T}$. Taking α as 15° the least number of teeth that can be used is $\frac{180}{15} = 12$ teeth, which is taken as the smallest standard interchangeable gear. Since there cannot be a fractional tooth on a gear, if the number of teeth does not come out a whole number, the next higher whole number must be taken.

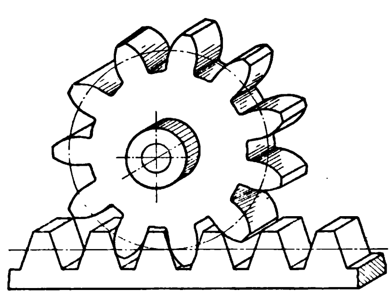


FIG. 93.

92. Involute Rack and Pinion.—If one of the spur gears of a pair is enlarged until its diameter is infinite, the combination will be that of a rack and pinion, shown in Fig. 93, since a rack is the same as a spur gear of infinite diameter.

Let AP, Fig. 94, be the radius of the pitch circle of the pinion

in a rack and pinion combination. The pitch circle of the rack being of infinite diameter is therefore a straight line.

Let ab be the line of obliquity, then Aa will be the radius of the base circle of the pinion.

If the pinion is the driver and turning as shown, contact cannot begin before the point a, the tangent point of the pinion's base circle with the line of obliquity, is reached, thus limiting the length of the rack teeth. Since the base line of the rack is

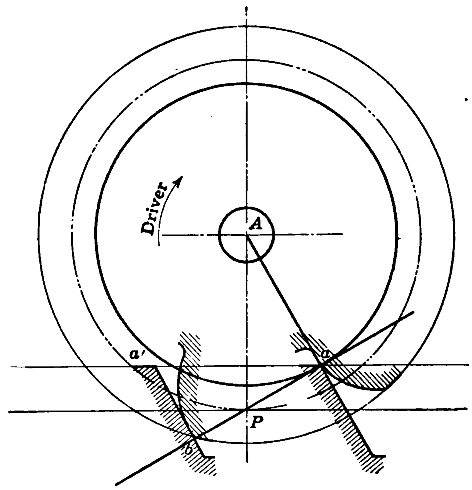


Fig. 94.

tangent to the line of obliquity at infinity, the length of the pinion teeth can theoretically be of infinite length, but practically they are *limited by their becoming pointed.*

A point on the line ab will generate the rack and pinion teeth, as n the case of spur gears (Art. 88). As the pinion turns in the direction of the arrow, the rack will move to the left with the same linear velocity as a point on the pitch circle of the pinion, which is greater than the linear velocity of a point on the base circle of the pinion in the ratio: $\frac{AP}{Aa}$.

When the generating point reaches b, the point a on the rack will have reached a', for in the triangles PAa and aba', ab is perpendicular to Aa and aa' is perpendicular to AP. Also $\frac{aa'}{ab} = \frac{AP}{Aa}$ so that the triangles are similar and the angle $a'ba$ which is equal to the angle AaP, is a right angle. Therefore

the outline of the rack tooth is a straight line perpendicular to the line of obliquity.

It will be noted that the directional relation between the rack and pinion is not constant, but depends upon the length of the rack.

93. Involute Annular Gear and Pinion.—An annular gear is one in which the teeth are cut internally on the pitch circle instead of externally as in the case of spur gears. Fig. 95 illustrates an annular gear and pinion combination.

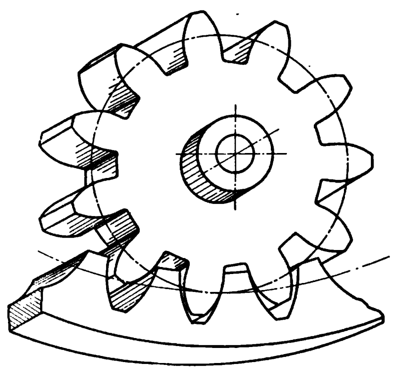


FIG. 95.

The method of laying out the teeth of the annular gear is precisely the same as for the spur gear, except that the tangent points of the base circles with the line of obliquity, lie on the same side of the pitch point instead of on opposite sides as in spur gears.

Let AP and BP, Fig. 96, be the pitch radii of the pinion and annular gear respectively, and let aP be the line of obliquity. Then the radii of the base circles will be Aa and Bb.

Let the pinion drive as shown, then since contact always begins between the drivers flank and followers face, and cannot begin before the tangent point of the pinions base circle with the line of obliquity is reached, the teeth of the annular gear cannot pass inside the point a. Ba will then be the shortest radius for the addendum circle of the annular gear.

The length of the pinion teeth, as in the case of the rack and pinion combination, are limited only by their becoming pointed.

In the figure, contact begins where the addendum circle of the

driven cuts the line of obliquity and ceases where the addendum circle of the driver cuts the line of obliquity.

It will be noted that the sides of the teeth on the annular gear are concave instead of convex, as in the case of spur gears. The annular gear tooth is exactly the same as the space between the teeth of a spur gear of the same angle of obliquity, pitch and number of teeth, with the exception of clearance top and bottom.

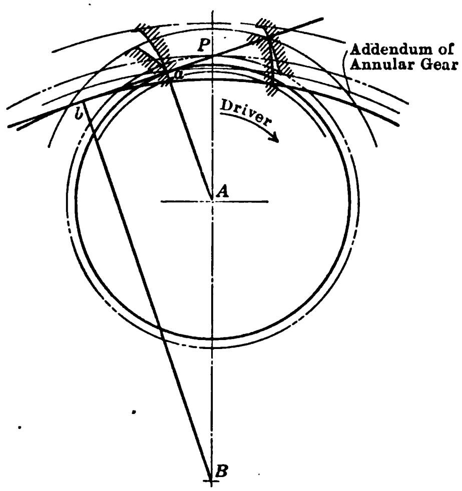


FIG. 96.

94. Interference of Involute Teeth.—If the teeth of the gears of Fig, 92 are long enough to pass outside of the points *a* and *b* there will be interference between this extra length of the teeth and that part of the flank of its mate, lying inside the base circle, unless the flank is hollowed out, or the extra length of face rounded off. The latter method is generally used as the objection to hollowing out the flank is that it weakens the tooth. This part of the tooth lying between the base and working depth circles is generally made radial.

95. Effect of Changing the Distance between Centers of Involute Gears.—In involute gears, the tooth curves being developed by a point on a line tangent to the base circles, if the centers of the gears are drawn apart, the same involutes will be generated and the gears will still transmit a constant velocity ratio, even though the pitch circles are not tangent.

The gears can be drawn apart until the teeth just mate at their ends, but in this case the backlash will be excessive.

The centers may also be moved toward each other so that the pitch circles overlap, the limit being when the backlash is reduced to zero.

This is a desirable property of involute gears as it allows gears to be used on shafts that are not the exact center distances apart, as calculated by the pitch diameters.

The path of action being along a straight line, the pressure on the bearings is constant and tends to beep the centers as far apart as possible, and prevent rattling if there is any looseness in the bearings. This property of involute gears used to be considered objectionable but with gears and bearings of modern design the pressure is not excessive.

96. Interchangeable Involute Gears.—In order that involute gears may be interchangeable, that is, any gear of a set mesh properly with any other gear, the pitch and angle of obliquity of one must be the same as the pitch and angle of obliquity of the mating gear.

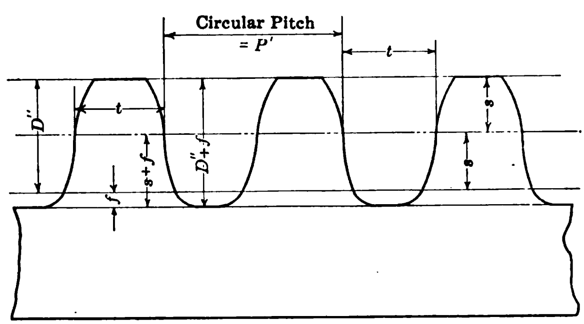


FIG. 97.

97. Standard Sizes of Teeth for Gears.—In the spur gears thus far discussed, nothing has been said about standard sizes of tooth parts. Two gears can be made to mesh properly together even though some of the parts are not the same size. For different pitches there are standard lengths and thicknesses of teeth however, and those adopted by the Brown and Sharpe Manufacturing Company and given in Tables A and B on pages 94 and 95 are almost universally used in this country. Fig. 97 shows parts of the teeth referred to in each column of the tables.

TABLE A

Gear Wheels

Table of Tooth Parts—Diametral Pitch in First Column

Diametral pitch P	Circular pitch P'	Thickness of tooth on pitch line t	Addendum or $\frac{1''}{P}$ s	Working depth of tooth D''	Depth of space below pitch line $s+f$	Whole depth of tooth $D''+f$
½	6.2832	3.1416	2.0000	4.0000	2.3142	4.3142
¾	4.1888	2.0944	1.3333	2.6666	1.5428	2.8761
1	3.1416	1.5708	1.0000	2.0000	1.5171	2.1571
1¼	2.5133	1.2566	.8000	1.6000	.9257	1.7257
1½	2.0944	1.0472	.6666	1.3333	.7714	1.4381
1¾	1.7952	.8976	.5714	1.1429	.6612	1.2326
2	1.5708	.7854	.5000	1.0000	.5785	1.0785
2¼	1.3963	.6981	.4444	.8888	.5143	.9587
2½	1.2566	.6283	.4000	.8000	.4628	.8628
2¾	1.1424	.5712	.3636	.7273	.4208	.7844
3	1.0472	.5236	.3333	.6666	.3857	.7190
3½	.8976	.4488	.2857	.5714	.3306	.6163
4	.7854	.3927	.2500	.5000	.2893	.5393
5	.6283	.3142	.2000	.4000	.2314	.4314
6	.5236	.2618	.1666	.3333	.1928	.3595
7	.4488	.2244	.1429	.2857	.1653	.3081
8	.3927	.1963	.1250	.2500	.1446	.2696
9	.3491	.1745	.1111	.2222	.1286	.2397
10	.3142	.1571	.1000	.2000	.1157	.2157
11	.2856	.1428	.0909	.1818	.1052	.1961
12	.2618	.1309	.0833	.1666	.0964	.1798
13	.2417	.1208	.0769	.1538	.0890	.1659
14	.2244	.1122	.0714	.1429	.0826	.1541
15	.2094	.1047	.0666	.1333	.0771	.1438
16	.1963	.0982	.0625	.1250	.0723	.1348
17	.1848	.0924	.0588	.1176	.0681	.1269
18	.1745	.0873	.0555	.1111	.0643	.1198
19	.1653	.0827	.0526	.1053	.0609	.1135
20	.1571	.0785	.0500	.1000	.0579	.1079
22	.1428	.0714	.0455	.0909	.0526	.0980
24	.1309	.0654	.0417	.0833	.0482	.0898
26	.1208	.0604	.0385	.0769	.0445	.0829
28	.1122	.0561	.0357	.0714	.0413	.0770
30	.1047	.0524	.0333	.0666	.0386	.0719

TABLE B

GEAR WHEELS

Table of Tooth Parts—Circular Pitch in First Column

Circular pitch P'	Threads or teeth per inch $\frac{1''}{P'}$	Diametral pitch P	Thickness of tooth on pitch line t	Addendum or $\frac{1''}{P}$ s	Working depth of tooth D''	Depth of space below pitch line $s+f$	Whole depth of tooth $D''+f$
2	$\frac{1}{2}$	1.5708	1.0000	.6366	1.2732	.7366	1.3732
$1\frac{7}{8}$	$\frac{8}{15}$	1.6755	.9375	.5968	1.1937	.6906	1.2874
$1\frac{3}{4}$	$\frac{4}{7}$	1.7952	.8740	.5570	1.1141	.6445	1.2016
$1\frac{5}{8}$	$\frac{8}{13}$	1.9333	.8150	.5173	1.0345	.5985	1.1158
$1\frac{1}{2}$	$\frac{2}{3}$	2.0944	.7500	.4775	.9549	.5525	1.0299
$1\frac{7}{16}$	$\frac{16}{23}$	2.1855	.7187	.4576	.9151	.5294	.9870
$1\frac{3}{8}$	$\frac{8}{11}$	2.2848	.6875	.4377	.8754	.5064	.9441
$1\frac{1}{3}$	$\frac{3}{4}$	2.3562	.6666	.4244	.8488	.4910	.9154
$1\frac{5}{16}$	$\frac{16}{21}$	2.3936	.6562	.4178	.8356	.4834	.9012
$1\frac{1}{4}$	$\frac{4}{5}$	2.5133	.6250	.3979	.7958	.4604	.8583
$1\frac{3}{16}$	$\frac{16}{19}$	2.6456	.5937	.3780	.7560	.4374	.8156
$1\frac{1}{8}$	$\frac{8}{9}$	2.7925	.5625	.3581	.7162	.4143	.7724
$1\frac{1}{16}$	$\frac{16}{17}$	2.9568	.5312	.3382	.6764	.3913	.7295
1	1	3.1416	.5000	.3183	.6366	.3683	.6866
$\frac{15}{16}$	$1\frac{1}{15}$	3.3510	.4687	.2984	.5968	.3453	.6437
$\frac{7}{8}$	$1\frac{1}{7}$	3.5904	.4375	.2785	.5570	.3223	.6007
$\frac{13}{16}$	$1\frac{3}{13}$	3.8666	.4062	.2586	.5173	.2993	.5579
$\frac{4}{5}$	$1\frac{1}{4}$	3.9270	.4000	.2546	.5092	.2946	.5492
$\frac{3}{4}$	$1\frac{1}{3}$	4.1888	.3750	.2387	.4775	.2762	.5150
$\frac{11}{16}$	$1\frac{5}{11}$	4.5696	.3437	.2189	.4377	.2532	.4720
$\frac{2}{3}$	$1\frac{1}{2}$	4.7124	.3333	.2122	.4244	.2455	.4577
$\frac{5}{8}$	$1\frac{3}{5}$	5.0265	.3125	.1989	.3779	.2301	.4291
$\frac{3}{5}$	$1\frac{2}{3}$	5.2360	.3000	.1910	.3820	.2210	.4120
$\frac{4}{7}$	$1\frac{3}{4}$	5.4978	.2857	.1819	.3638	.2105	.3923
$\frac{9}{16}$	$1\frac{7}{9}$	5.5851	.2812	.1790	.3581	.2071	.3862
$\frac{1}{2}$	2	6.2832	.2500	.1592	.3183	.1842	.3433
$\frac{4}{9}$	$2\frac{1}{4}$	7.0685	.2222	.1415	.2830	.1637	.3052
$\frac{7}{16}$	$2\frac{2}{7}$	7.1808	.2187	.1393	.2785	.1611	.3003
$\frac{3}{7}$	$2\frac{1}{3}$	7.3304	.2143	.1364	.2728	.1578	.2942
$\frac{2}{5}$	$2\frac{1}{2}$	7.8540	.2000	.1273	.2546	.1473	.2746
$\frac{3}{8}$	$2\frac{2}{3}$	8.3776	.1875	.1194	.2387	.1381	.2575
$\frac{4}{11}$	$2\frac{3}{4}$	8.6394	.1818	.1158	.2316	.1340	.2498
$\frac{1}{3}$	3	9.4248	.1666	.1061	.2122	.1228	.2289
$\frac{5}{16}$	$3\frac{1}{5}$	10.0531	1562	.0995	.1989	.1151	.2146
$\frac{3}{10}$	$3\frac{1}{3}$	10.4719	.1500	.0955	.1910	.1105	.2060
$\frac{2}{7}$	$3\frac{1}{2}$	10.9956	.1429	.0909	.1819	.1052	.1962

The sizes of the parts apply to other systems of teeth as well as the involute system.

98. To Lay Out a Pair of Standard Involute Spur Gears.

Problem.—Lay out the tooth curves for a pair of involute spur gears of 24 teeth and 16 teeth respectively; 2 diametral pitch and 15° angle of obliquity.

24 teeth, 2 diametral pitch = 12 in. pitch diameter for large gear.
16 teeth, 2 diametral pitch = 8 in. pitch diameter for small gear.
Draw the pitch circles Fig. 98 with radii AP and BP equal to

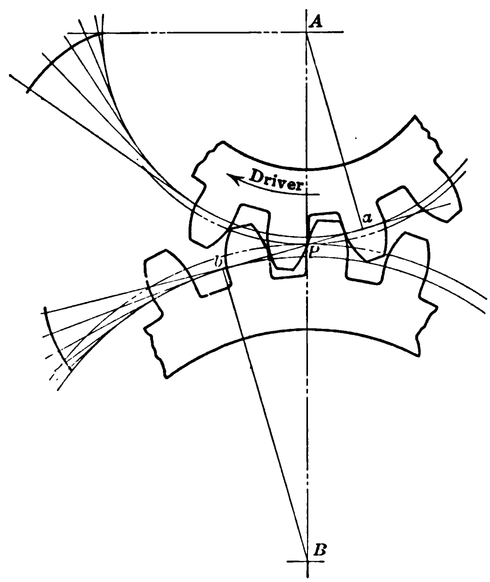


Fig. 98.

4 in. and 6 in. respectively, and through P draw the line of obliquity ab having the given angle of obliquity with the common tangent to the pitch circles through the point P. Drop perpendiculars from the centers A and B, cutting the line of obliquity in a and b respectively. Then Aa and Bb will be the radii of the base circles which can now be drawn.

From Table A on page 94 find the amount that the addendum, dedendum and working depth circles differ from the pitch circle, and draw them in.

Divide the pitch circle of the smaller gear into 16 equal parts and that of the larger into 24 equal parts, which will give the

circular pitch. When no backlash is allowed, the thickness of the tooth, and the width of space measured on the pitch circle will be the same. Bisect the circular pitch on each of the gears which will give 32 equal divisions on the pitch circle of the small gear and 48 on the larger one.

At any point on the base circle of each gear develop an involute, and draw in the curves between the base and addendum circles through alternate points on the pitch circles. This will give one side of all the teeth on each gear. The curve for the other side of the teeth is the reverse of the one just drawn.[1] Draw the part of the teeth lying between the base and working depth circles radial, and put in a small arc or fillet between the working depth and dedendum circles. This fillet should be as large as possible without passing outside of the working depth circle.

PROBLEMS

44. Make a sketch of a pair of involute spur gears, assuming the angle of obliquity and diameters of the pitch and addendum circles. Show the path of contact and skeleton teeth where contact begins and ceases only.

45. Make a sketch of two involute spur gears of unequal diameter, with the smaller gear driving clockwise. Assume the angle of obliquity and the addendum circles so that there will be no angle of approach, but a maximum angle of recess.

46. Make a sketch showing the path of contact in a rack and pinion combination, involute system, with the rack driving to the left. Assume the angle of obliquity and addendum lines.

47. Make a sketch showing the path of contact in an annular gear and pinion combination, involute system, with the pinion driving counter clockwise. Assume the angle of obliquity and the diameters of addendum circles.

48. Draw the correct tooth curves for two spur gears that are in mesh. Involute system. Cut templets to lay out the tooth curves and check them for contact. Show the teeth going into and out of action.

[1] A convenient method of transferring the original involute to draw in the tooth curves is by means of a templet. This can be made of a thin piece of soft wood. Put a fine needle through the wood, near one end, and the center of the gear, then cut away the wood to conform to the developed tooth outline. Then swinging the templet about the center draw in one side of all the teeth. Draw the other side of the teeth by turning the templet over and keeping the same centers. The same templet can be used for two gears only when they have the same number of teeth and angle of obliquity.

Data: Number of teeth in gears 36 and 21.
Diametral pitch $1\frac{1}{2}$; angle of obliquity 15°.
Small gear is driver and revolves counter-clockwise.
Thickness of rim below tooth bottoms 2 in.

Note.—Make center line of gears horizontal and in center of sheet with smaller gear to the right.

Lay out the pitch, addendum, dedendum, base and working depth circles, and develop the involute between the base and addendum circles. The part of the tooth between the base and working depth circles is to be drawn radial. Cut templets to fit the tooth outline between the *working depth* and *addendum* circles. Space out the teeth on the pitch circles. Ink drawing, slant lettering. Time, 5 hours.

49. Draw the correct tooth curves for an involute rack and pinion that are in mesh. Check the templet for contact and show the teeth going into and out of action.

Data: Teeth in pinion 21. Diametral pitch $1\frac{1}{2}$. Angle of obliquity 15°. Pinion is driver and turns clockwise.

Note.—Make rack horizontal and center of pinion in center of sheet right and left and $1\frac{1}{4}$in. from top border line. Use templet of 21 tooth gear of problem 48. No statement of problem to go on sheet, but under the drawing put INVOLUTE RACK & PINION in vertical capital letters. Ink drawing. Time, 3 hours.

50. Draw the correct tooth curves for an involute annular gear and pinion that are in mesh.

Data: Same as problem 48. Use same templets.

Note.—No statement of problem or data. Only INVOLUTE ANNULAR GEAR & PINION. Ink drawing. Time, 3 hours.

CYCLOIDAL SYSTEM

99. The cycloidal system, as the name implies has tooth curves of cycloidal form. It is an older system than the involute, but is being replaced by it for many classes of work.

Before taking up the tooth outlines, a brief discussion of the construction of the different cycloidal curves will be given.

100. Cycloid.—A cycloid is the curve generated by a point on the circumference of a circle rolling on a straight line. Let the circle of radius OT, Fig. 99, be the rolling or generating circle, which rolls on the straight line $T6'$.

Divide the semi-circumference of the circle up into any convenient number of equal parts as six, and lay off on the straight line $T1' = T1$, $1'2' = 12$, $2'3' = 23$, $3'4' = 34$, $4'5' = 45$ and $5'6' = 56$.

Through $1'$, $2'$, $3'$, $4'$, $5'$ and $6'$ draw perpendiculars to $T6'$, and through the points 1, 2, 3, 4 and 5 on the semi-circle draw lines parallel to $T6'$, intersecting the diameter of the semi-circle

in 1_1, 2_1, O, 4_1, and 5_1, and the perpendiculars in $1''$, $2''$, $3''$, $4''$, $5''$ and $6''$.

On 1_1, $1''$ from $1''$, lay off $1''a=11_1$; on $2_12''$ from $2''$ lay off $2''b=22_1$, etc. Through the points a, b, c, d, e, and $6''$ draw a smooth curve, which will be the cycloid traced by T as the circle rolls on the straight line.

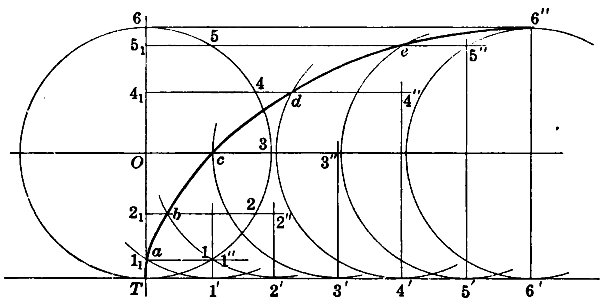


FIG. 99.

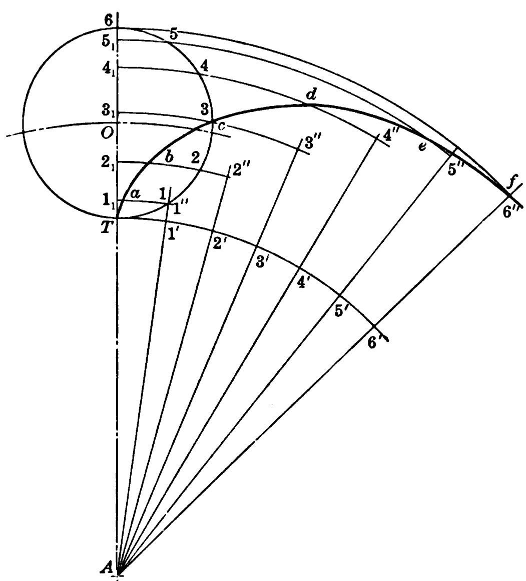


FIG. 100.

101. Epicycloid.—The epicycloid is the curve traced by a point on the circumference of a circle as it rolls on the outside of a second circle, called the directing circle. Let OT, Fig. 100,

be the radius of the rolling circle and AT, the radius of the directing circle.

Divide the semi-circumference of the rolling circle into any convenient number of equal parts as six, and lay off from T, divisions of the same length on the directing circle. Through the points $1'$, $2'$, $3'$, . . . $6'$ just found and through the center A draw lines.

With A as a center and radii $A1$, $A2$, $A3$, . . . $A6$ draw arcs intersecting the diameter of the rolling circle in 1_1, 2_1, 3_1, . . . 5_1, and the radial lines in $1''$, $2''$, $3''$ . . . $6''$. On $1''1_1$, from $1''$ lay off $1''a = 11_1$; on $2''2_1$, lay off $2''\,b = 22_1$; on $3''3_1$, lay off $3''c = 33_1$ etc.

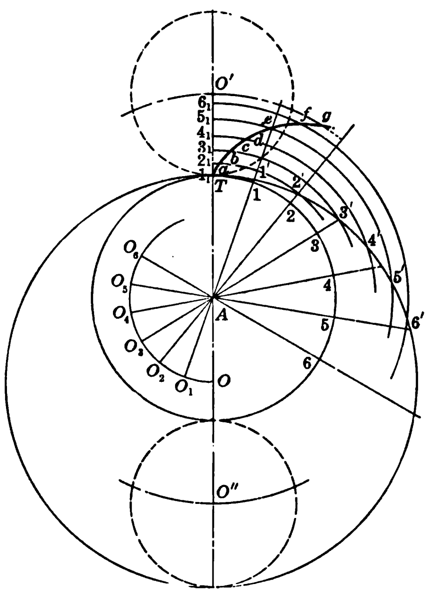


FIG. 101.

Though the points a, b, c, d, e and $6''$ draw a smooth curve which will be the cycloidal curve.

If the rolling circle is larger than the directing circle and rolled *internally* on it, an epicycloid will still be generated.

In Fig. 101 let AT be the radius of the directing circle and $O'T$ and OT the radii of the rolling circles which roll externally and internally on the directing circle respectively. In this case, the diameter of the large rolling circle is taken equal to the sum

of the diameters of the directing circle and small rolling circle. The curve generated by the large rolling circle rolling internally will then be the same as that generated by the small rolling circle rolling externally. This is called the double generation of the epicycycloid, and will be referred to later under annular gears.

102. Hypocycloid.—The hypocycloid is a curve generated by a point on the circumference of a circle rolling internally on the directing circle.

The construction is shown in Fig. 102, and is the same as that for the epicycloid, so that no further explanation is necessary.

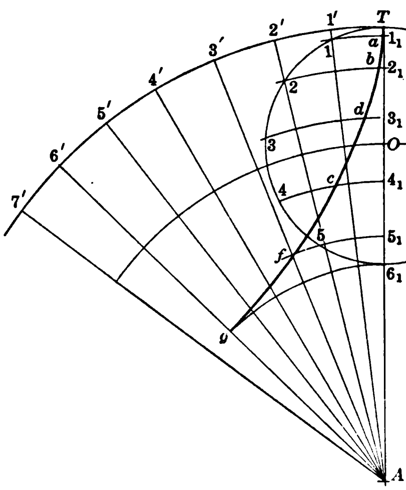


FIG. 102.

If the rolling circle has a diameter one-half that of the directing circle, the hypocyloid formed will be a radial line of the directing circle.

103. Application of Cycloidal Curves to Gear Teeth.—In Fig. 103 let AP be the radius of the pitch circle of a gear, and DP the radius of a circle that rolls on the outside of the pitch circle and traces the epicycloid Pm. CP is the radius of a circle that rolls on the inside of the pitch circle and traces the hypocycloid Pn. Pm will be the *face* and Pn the *flank* of a gear tooth which is shown shaded.

104. Cycloidal Spur Gears.—Let AP and BP, Fig. 104, be the pitch radii and CP and DP the radii of the rolling circles of two cycloidal spur gears that are in mesh.

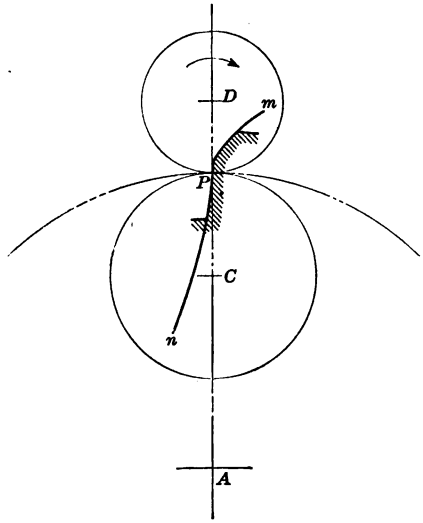


Fig. 103.

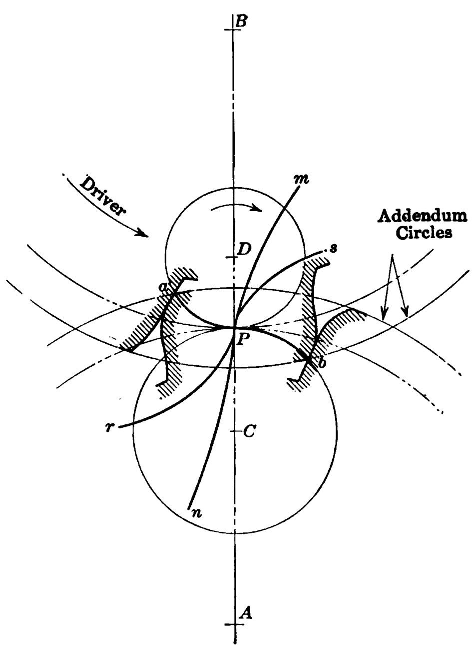


Fig. 104.

The rolling circle with its center at C, when rolled on the inside of the lower pitch circle generates the hypocycloid Pn, and when rolled on the outside of the upper pitch circle generates the epicycloid Pr. These curves are the flank of the lower gear and the face of the upper gear respectively, and are generated simultaneously.

The rolling circle with a center at D when rolled on the inside of the upper pitch circle generates the hypocycloid Pm, and when rolled on the outside of the lower pitch circle generates the epicycloid Ps. These curves are respectively the flank and face of the teeth of B and A, and are also generated simultaneously.

The tooth outlines that are generated simultaneously are the parts of the teeth that are in contact, which is the face of one tooth with the flank of its mate on the other gear.

If the addendum circles are as shown and the upper gear is the driver turning in a counter clockwise direction, contact will begin at a where the addendum circle of the driven cuts the rolling circle of the driver. It will follow along the upper rolling circle until the pitch point is reached. At this point contact will change from the drivers *flank* and followers *face*, to the drivers *face* and followers *flank* where it will follow along the rolling circle of the driven to the point b where the addendum circle of the driver cuts the rolling circle of the driven, where contact between that pair of teeth will cease.

From the figure it can be seen that the length of the face of the driver's teeth governs the angle of recess, and the length of the face of the driven gear's teeth governs the angle of approach.

Thus, if no angle of approach is desired, the teeth of the driven gear will have no faces, and the driver's teeth will require no flanks. This case is sometimes used in the case of the rack and pinion on planer beds.

It will also be noted that if the diameters of the rolling circles are made equal to the pitch radii of their respective gears, that the flanks of the teeth will be radial.

105. Cycloidal Rack and Pinion.—In the rack and pinion combination shown in Fig. 105 let AP be the radius of the pitch circle of the pinion, and C and D the centers of the rolling circles.

The upper rolling circle when rolled on the inside of the pitch circle of the pinion generates the hypocycloid Pm, the flank of the pinion teeth, and when rolled on the pitch line of the rack generates the cycloid Ps, the face of the rack teeth. The lower

rolling circle when rolled on the outside of the pinion's pitch circle generates the epicycloid *Pr*, the face of the pinion teeth, and generates the flank of the rack teeth when rolled on the lower side of the pitch line of the rack, which is the cycloid *Pn*.

If the pinion drives clockwise as shown, contact will begin at *a* on the rolling circle of the pinion, and cease at *b* on the rolling circle of the rack.

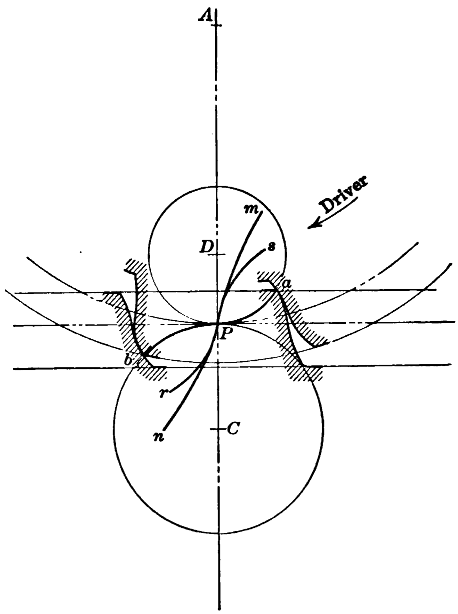


FIG. 105.

106. Cycloidal Rack Teeth with Straight Flanks.—In order to have straight flanks on the rack teeth, the rolling circle of the rack will be of infinite diameter and will coincide with the pitch line. The faces of the pinion teeth, which are developed by this circle, will therefore be involutes. The disadvantage of the form of tooth is that all of the wear on the rack teeth is between the pitch and addendum lines, there being no contact on the rack teeth below the pitch line.

107. Cycloidal Annular Gear and Pinion.—In Fig. 106 let *AP* be the pitch radius of the pinion and *BP* the pitch radius of the annular gear in an annular gear and pinion combination. Let *PC* be the radius of the inside rolling circle, and *PD* the

radius of the outside rolling circle. The outside rolling circle will generate the epicycloids *Pm* and *Pn* which are the flank of the annular gear teeth and the face of the pinion teeth respectively. The face of the annular gear tooth is the hypocycloid *Pr*, while the flank of the pinion tooth is the hypocycloid *Ps*. These curves are generated by the inside rolling circle.

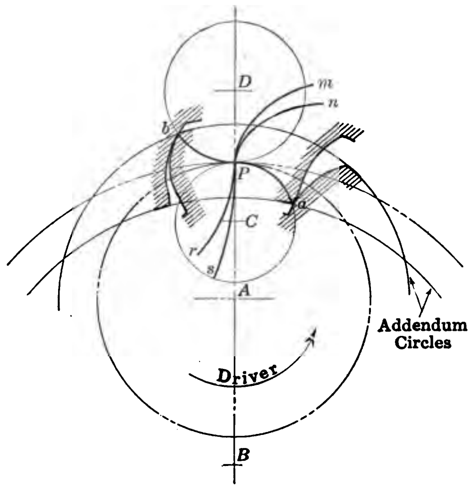


FIG. 106.

With the pinion driving as shown, contact will begin at *a* and cease at *b*. The method of laying out the gears is in every respect similar to that of spur gears.

108. Limiting Size of Annular Gear Pinion.—It was seen in Art. 101 that the same epicycloid could be generated by two different sized rolling circles. The difference in their diameters being equal to the diameter of the directing circle. On account of this fact, the difference between the pitch circles of a cycloidal annular gear and pinion combination must be equal to the sum of the diameters of the rolling circles or else there will be what is called secondary contact between the teeth.[1]

If the pitch circles differ by the sum of the diameters of the rolling circles, and the gears are standard, interchangeable gears, they will differ by twelve teeth.

[1] For a full discussion of this property of the cycloidal annular gear and pinion see Professor MacCord's "Kinematics," also a "Treatise on Gear Wheels," by George B. Grant, M. E.

Provided one of the gears have no faces on its teeth, they need differ by but six teeth.[1]

109. Interchangeable Cycloidal Gears.—In the different pairs of cycloidal gears thus far considered, the rolling circles have not been taken of the same diameter, and it is possible to lay out the teeth of two gears having the same pitch that will mesh properly together even though their rolling circles are not the same size. If, however, the gears are to be interchangeable, that is, any gear mesh with any other of the set, they must, besides having the same pitch, have their tooth curves developed by the same size rolling circles. In Fig. 107 if

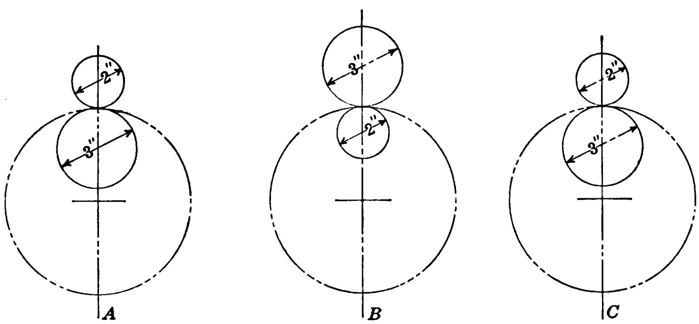


FIG. 107.

the gears *A*, *B* and *C* are all of the same pitch, and their tooth curves developed by rolling circles of the diameters shown, *B* will mesh properly with *A* and *C*, but *A* and *C* will not mesh with each other.

110. Standard Diameter of Rolling Circle for Cycloidal Gears.—The standard diameter of rolling circle is one that will give radial flanks on a gear of twelve teeth. In order to have radial flanks on the teeth, the diameter of the rolling circle must be one-half that of the pitch circle. (Art. 102).

This does not mean that all the gears of the set will have radial flanks, but only the 12-tooth gear.

For example, the standard diameter rolling circle for a 64-tooth 4-pitch gear would be the same as that for a 12-tooth of the same pitch, or $\frac{12}{4}=3$-in. pitch diameter, and $\frac{3}{2}$ in. $=1\frac{1}{2}$-in. diameter of rolling circle.

Geo. B. Grant says:[2] "The standard adopted by the manu-

[1] The reason for this is discussed in Art. 110.

[2] Treatise on Gear Wheels, page 41.

facturers of cycloidal gear cutters is that having radial flanks on a gear of fifteen teeth, but is not and should not be in use for other purposes. If any change is made, it should be in the other direction, to make the set take in gears of ten teeth.

"It must be borne in mind that the standard adopted does not limit the set to the stated minimum number of teeth, but that it simply requires that the smaller gears have weak under curved teeth."

111. Comparison of Involute and Cycloidal Systems.—In the two systems, the advantages are practically all in favor of the involute tooth. Some of them are as follows:

1. The distance between the centers can be changed without affecting the velocity ratio.
2. The path of contact is a straight line, thus keeping a constant pressure on the axes.
3. Tooth curves of single curvature.
4. Involute rack teeth have straight sides, thus cutter is easily made.
5. Wear on involute teeth more uniform than in cycloidal.
6. Less cutters required to cut a complete set of gears.

The cycloidal system is the older of the two, and for those using it, the expense of changing over to the involute system would be large, but the main objection would be that for repairs on machines already equipped with cycloidal gears, cutters of that system must be kept on hand.

PROBLEMS

51. Construct an epicycloid for a 3 in. generating circle and an 8 in. directing circle.

52. Construct hypocycloids using a 12 in. directing circle and generating circles of 4 in. and 7 in. respectively.

53. In a cycloidal spur gear combination of unequal size, assume the diameters of the pitch, addendum and rolling circles, and show the path of contact for the small gear driving clockwise.

54. In a cycloidal rack and pinion combination with the pinion driving counter-clockwise, assume the diameters of pitch, addendum and rolling circles so that there will be no angle of recess, and show path of contact.

55. What are the standard diameters of rolling circles that would be used to generate the tooth outlines for each of the rollowing gears?

1. 28 teeth 2 pitch	4. 86 teeth 4 pitch.
2. 49 teeth 2 pitch.	5. 4 pitch 20 in. pitch diameter.
3. 100 teeth 12½ in. pitch diameter.	6. 50 teeth 1 in. pitch.

56. Draw the correct tooth curves for two spur gears that are in mesh, cycloidal system. Cut templets to lay out the tooth curves and check them for contact. Show the teeth going into and out of action.

Data: Number of teeth in gears 36 and 21.
Diametral pitch $1\frac{1}{2}$.
Diameter of rolling circles 4 in.
Thickness of rim below tooth bottoms 2 in.
Small gear to drive and revolve clockwise.

Note.—Make center line of gears horizontal and in center of the sheet with the smaller gear to the right. Layout the pitch, addendum, dedendum and rolling circles. Develop the tooth curves and cut templets to fit them. Space out the teeth on the pitch circles. Ink drawing. Time, 5 hours.

57. Draw the correct tooth curves for a cycloidal rack and pinion that are in mesh. Check the templets for contact and show the teeth going into and out of contact.

Data: Teeth in pinion 21.
Diametral pitch $1\frac{1}{2}$.
Diameter of rolling circles 4 in.

Note.—Make rack horizontal and center of pinion in center of sheet right and left and $1\frac{1}{4}$ in. from top border line. Pinion to drive counter-clockwise. Use templet of 21-tooth gear of problem 56. No statement of problem but this title, CYCLOIDAL RACK & PINION. Time, 4 hours.

58. Draw the correct tooth curves for a cycloidal annular gear and pinion that are in mesh.

Data: Same as for problem 56. Use same templets. No statement of problem or data, but this title, CYCLOIDAL ANNULAR GEAR & PINION.
Time, 3 hours.

PIN GEARS

112. Pin Gearing.—If the diameter of the rolling circle having its center at D, Fig. 104, is increased until it coincides with its pitch circle it will generate epicycloids for the faces of the teeth of A and a point for the teeth of B.

Since a point or pin tooth could do no work, it is usual to substitute a pin of sensible diameter, then the tooth curve on the other gear will be parallel to the true epicycloid, and a distance away from it equal to the radius of the pin. In using pin gears, a small number of pins can be used on a gear to correspond to a small number of teeth, but they have not the disadvantage of having weak teeth. Fig. 108 shows a pin wheel of six teeth meshing with a toothed gear. The dotted outline shows the teeth of gear if the pins had no diameter. The circular pitch must of course be the same in each gear, or the arc $P1'$ must equal the arc $P1$. The maximum radius of addendum circle is

found by drawing a straight line $P1$ and finding where it cuts the circumference of the pin at a. The radius of the addendum circle will be from the center of the gear to a.

Sometimes rolls are placed on the pins in order to make the action between the teeth more nearly a rolling one. In this case the pins are secured between two circular plates, and the gear is then known as a *lantern wheel.* This type is common in watch and clock mechanisms, except that no rolls are used.

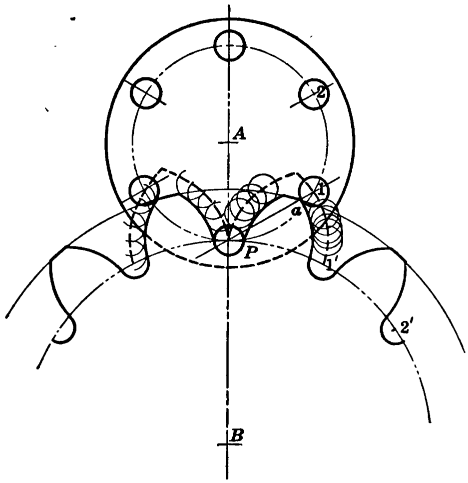


FIG. 108.

The pins should always be on the driven wheel since then the action between the teeth is almost all on the recess side. It will be wholly so if the pins have no diameter.

Pin gears are not confined to spur gears, but can be used in any combination, and the pins can be on either gear, as a pin annular gear and a toothed pinion.

Stepped and Helical Gears

113. Stepped and Helical Gears.—Smoother action can be obtained in a pair of gears by having more teeth in contact at one time. This can be done by lengthening the teeth or by making the pitch smaller. By lengthening the teeth, the angle of action is made greater and since more teeth are brought into contact at one time, the pressure on each tooth is decreased, but

gear teeth are assumed to act as cantilever beams and the teeth are lengthened at a greater rate than the number of pairs of teeth in contact is increased, so that the resulting gear is weaker.

By decreasing the pitch the thickness of the tooth is decreased, and this is at a greater rate than the pairs of teeth are increased, so again the gear is made weaker.

More pairs of teeth can be brought into contact at one time by placing several gears side by side on the same shaft, and by advancing the teeth of each ahead of the one next to it, by the circular pitch divided by the number of gears, and then fastening them securely together. Such a gear is called a *stepped gear*.

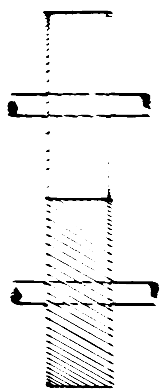


Fig. 109.

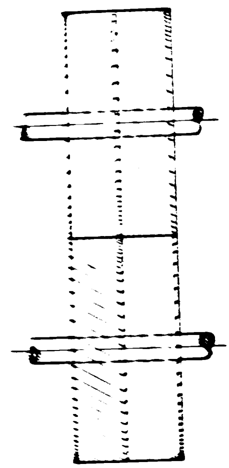


Fig. 110.

If the number of steps is increased indefinitely, and the teeth on each placed $\frac{1}{n}$th of the circular pitch ahead of the one next to it, where n equals the number of steps, a line drawn through the center of the top of each tooth will be a helix, and such gears are called *helical, spiral** or *twisted gears*. A pair of such gears are shown conventionally in Fig. 109.

If the power to be transmitted is great, there will be con-

* The term "spiral" as applied to gears suggests a gear the teeth of which lie in a plane, but the word is used to denote gears the teeth of which have helical form, and while it has common usage the terms twisted or helical would perhaps convey a better idea of the gear. This is suggested by Mr. R. E. Flanders in his book on "Gear Cutting Machinery."

siderable end thrust, that is, one gear will tend to slide by the other along its shaft. This tendency is overcome by placing on the shaft gears of the same size and of equal but opposite helix angles. These gears are represented conventionally in Fig. 110, and are known as "herring-bone" gears.

There has recently been introduced into this country a system of herring-bone gears known as the Wuest gear, named after its inventor. The gear is cut from a solid piece and the teeth of one half are advanced one-half the circular pitch ahead of the other.[1]

Approximate Methods of Laying out Tooth Curves

114. There are a number of methods of laying out tooth curves by using templets, rectangular coördinates and circular arcs. It is not necessary to use these methods as much since gear cutting machinery has been brought to such a high state of development as it was when the pattern maker or millwright had to lay out the teeth of gears that were usually cast.

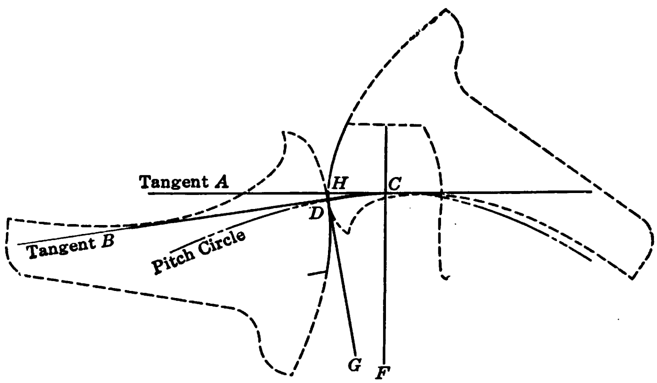


Fig. 111.

The Robinson Templet Odontograph is one of the best known templet methods for laying out tooth curves. The odontograph, together with the method of forming the side of a tooth, is illustrated in Fig. 111. It is necessary to have a table to set the instrument properly.

[1] See article on "The Herring-bone Gear" by P. C. Day. *Journal of the American Society of Mechanical Engineers,* for Jan., 1912.

The curves of the sides of the templet are logarithmic spirals, one being the evolute of the other.

The holes in the templet are for the purpose of fastening it to a bar, so that it can be swung about the center of the gear for the purpose of drawing in all of the tooth outlines. It will be noticed that two settings of the instrument are necessary to draw in one side of a tooth. The Willis Odontograph is illustrated in Fig. 112 and is for laying out involute teeth by the method of circular

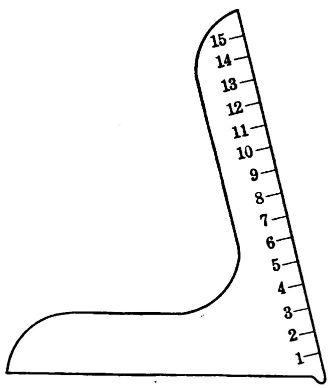


Fig. 112.

arcs. The two legs of the templet make an angle of 90° minus the angle of obliquity with each other. The setting for the radius of the tooth outline depends upon the pitch diameter of the gear, and after the setting is obtained, the teeth can be drawn in.

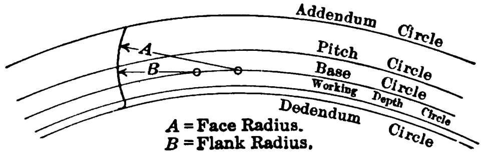


Fig. 113.

Grant's Involute Odontograph is also a method of laying out involute teeth by means of circular arcs. The lengths of radii to be used for different pitches and numbers of teeth can be found from Table C.

The method of laying out the radii is shown in Fig. 113.

TABLE C.—GRANT'S INVOLUTE ODONTOGRAPH

Standard Interchangeable Teeth

Centers on Base Line

Teeth	Divide by the diametrical pitch		Multiply by the circular pitch	
	Face radius	Flank radius	Face radius	Flank radius
10	2.28	.69	.73	.22
11	2.40	.83	.76	.27
12	2.51	.96	.80	.31
13	2.62	1.09	.83	.34
14	2.72	1.22	.87	.39
15	2.82	1.34	.90	.43
16	2.92	1.46	.93	.47
17	3.02	1.58	.96	.50
18	3.12	1.69	.99	.54
19	3.22	1.79	1.03	.57
20	3.32	1.89	1.06	.60
21	3.41	1.98	1.09	.63
22	3.49	2.06	1.11	.66
23	3.57	2.15	1.13	.69
24	3.64	2.24	1.16	.71
25	3.71	2.33	1.18	.74
26	3.78	2.42	1.20	.77
27	3.85	2.50	1.23	.80
28	3.92	2.59	1.25	.82
29	3.99	2.67	1.27	.85
30	4.06	2.76	1.29	.88
31	4.13	2.85	1.31	.91
32	4.20	2.93	1.34	.93
33	4.27	3.01	1.36	.96
34	4.33	3.09	1.38	.99
35	4.39	3.16	1.39	1.01
36	4.45	3.23	1.41	1.03
37–40	4.20		1.34	
41–45	4.63		1.48	
46–51	5.06		1.61	
52–60	5.74		1.83	
61–70	6.52		2.07	
71–90	7.72		2.46	
91–120	9.78		3.11	
121–180	13.38		4.26	
181–360	21.62		6.88	

Gear Cutting

115. Cutting Spur and Annular Gears.—The general method for cutting these gears is to use a tool, the outline of the cutting face of which is the same as the space between two teeth. Fig. 114 shows a tool that can be used in a planer or shaper for

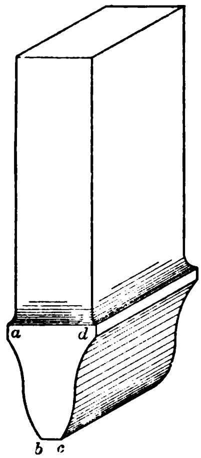


Fig. 114.

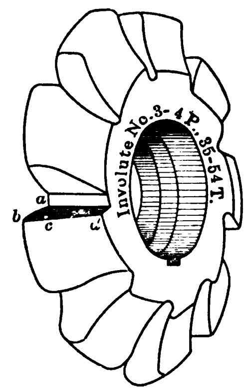


Fig. 115.

cutting spur and annular gears. The profile *abcd* is the shape of the space between the teeth. The tool can be sharpened across the front face without changing the profile.

Fig. 115 shows a circular cutter for use in a milling machine. This also has the profile *abcd* the shape of the space between the teeth, and the face can be sharpened parallel to the axis without changing the profile. This cutter cannot be used as readily for annular gears as that of Fig. 114.

116. Interchangeable Gear Cutters.—In order to use either of the two cutters just described to cut a gear theoretically correct, it is necessary to have a different cutter for each different number of teeth and pitch, since the space between the teeth changes with each number of teeth. Therefore to cut a complete set of gears of any one pitch, from twelve teeth to a rack theoretically correct, it will take as many different cutters as there are gears.

It has been found, however, that one cutter can be used to cut several different numbers of teeth of any one pitch, accurate

enough for practical purposes, as in the larger gears the space does not change very much by the addition of a few teeth.

The following table shows the cutters used by the Brown and Sharpe Mfg. Co. for gear teeth.

Involute cutters 8 cutters in each set		Cycloidal cutters 24 cutters in each set			
Cutter	Teeth	Cutter	Teeth	Cutter	Teeth
No. 1 cuts	135 to rack	A cuts	12	M cuts	27 to 29
No. 2 cuts	55 to 134	B cuts	13	N cuts	30 to 33
No. 3 cuts	35 to 54	C cuts	14	O cuts	34 to 37
No. 4 cuts	26 to 34	D cuts	15	P cuts	38 to 42
No. 5 cuts	21 to 25	E cuts	16	Q cuts	43 to 49
No. 6 cuts	17 to 20	F cuts	17	R cuts	50 to 59
No. 7 cuts	14 to 16	G cuts	18	S cuts	60 to 74
No. 8 cuts	12 to 13	H cuts	19	T cuts	75 to 99
		I cuts	20	U cuts	100 to 149
		J cuts	21 to 22	V cuts	150 to 249
		K cuts	23 to 24	W cuts	250 or more
		L cuts	25 to 26	X cuts	rack.

The cutters are given in terms of the number of teeth, for by this method they are applicable to all pitches.

It will be noticed that there are eight cutters necessary for a set of involute gears, while in the cycloidal system there are twenty-four cutters. The reason for this is that in the involute system the tooth outlines do not change as rapidly as in the cycloidal system.

117. Conjugate Methods of Cutting Teeth.—If instead of using cutters that have the shape of the *space* between the teeth, those having an outline the same as the *tooth* of a gear are used, it is possible to cut a complete set of gears of one pitch with a single cutter.

This is known as the molding-generating principle and is largely used for cutting spur gear teeth.

Imagine a gear blank of wax or other plastic material rolled with a gear or rack on which the teeth have been cut, their pitch circles being tangent and rolling together without slipping.

Teeth will be formed in the soft material of the blank, which will mesh properly with the molding gear.

The gear blank of course cannot be made of wax, so in order to get the same result, the molding gear has cutting edges on its teeth and is given a reciprocating motion across the face of the blank. The rolling of the pitch circles together taking place

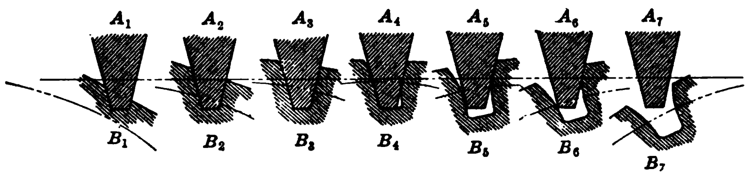


FIG. 116.

after the cutting gear has passed across the face of the blank and come back ready for the second stroke.

In Fig. 116 let A be a single tooth of an involute rack[1] and B a blank on which teeth are to be cut. Let the motion be such that the pitch line of the rack tooth and the pitch circle of the gear roll together without slipping, and also give the rack tooth, which has a cutting edge like the planer tool of Fig. 114, a

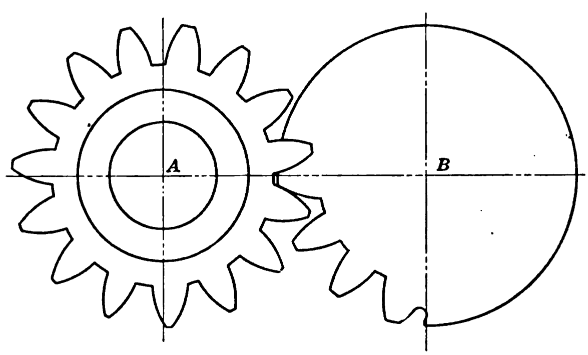


FIG. 117.

reciprocating motion across the face of the blank. The different positions represent the stages in the cutting of the space between two teeth.

All of the gears cut with this tooth will mesh properly with it and will also mesh correctly with each other.

[1] Any other involute or cycloidal gear tooth could be used equally well.

Fig. 117 shows a complete gear for a cutter instead of a single tooth. The gear has cutting edges the same as the single tooth, and is first fed in to the proper depth, that is so that the pitch circles are tangent, and then both rotate slightly before beginning each cutting stroke. Several teeth are in process of being cut at one time, although but one tooth is completed at a time.

118. Fellows Gear Shaper.—This machine, illustrated in Fig. 118, uses a cutter shaped like a pinion, and works on the molding-generating principle.

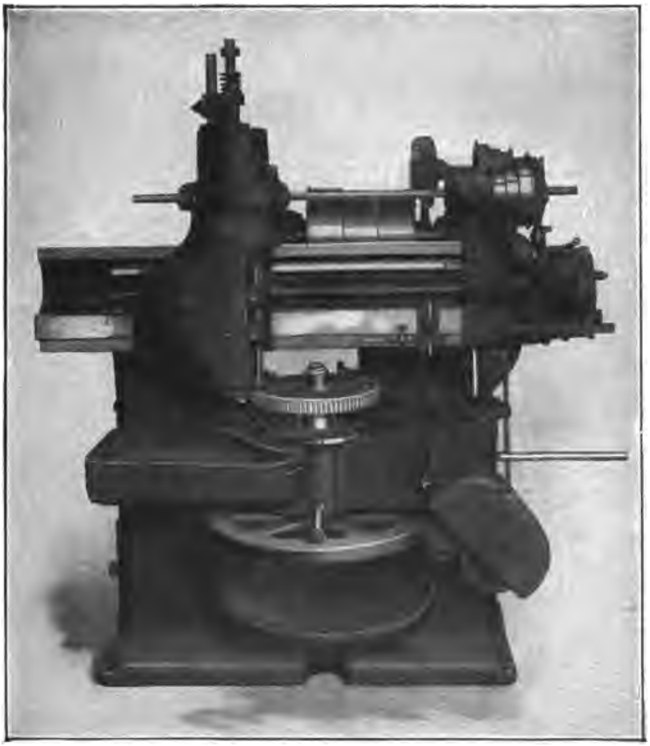


FIG. 118.

The cutter and blank are fastened to vertical axes which are connected so that when one rotates the other is rotated the proper amount, the two moving so that their pitch circles roll together without slipping. In addition to this, the spindle carrying the cutter can reciprocate in the direction along its

axis, so that when ascending the teeth of the cutter act as cutting tools on the blank in the same way as the cutting tool of a shaper.

The cutter spindle, which is carried on a cross-rail so that the axis of the cutter can be moved toward that of the blank.

The action of the machine is as follows: The cutter and blank are keyed to their respective axes, with neither the cutter nor the blank rotating.

During the up-stroke, the teeth of the cutter cut metal from the blank, and on the down-stroke the cutter-spindle is fed toward the blank axis a small amount.

This feeding in motion is continued at the end of each non-cutting stroke until the pitch circles are tangent and then ceases. In place of this, at the end of each non-cutting stroke, both the cutter and blank axes rotate slightly through corresponding angles so that their pitch circles roll together without slipping.

During the cutting stroke neither the cutter nor blank rotate. After the blank has made one complete revolution the teeth are all cut. Each tooth of the cutter acts as a cutting tool in turn, although several teeth of the blank are acted upon at one time.

One cutter will cut all gears of the same pitch, and all gears that are cut by the cutter will mesh properly with it, and with each other.

Either involute or cycloidal cutters can be used. An objection to the cycloidal form of cutter is that they must be produced by means of templets, while the involute form can actually be generated.

119. Gear Hobbing.—The gear hobbing machine is generally designed to cut teeth by the molding-generating principle in which a rack form of tooth is used for the cutter, instead of the circular cutter, but instead of a single rack tooth as in Fig. 116, a 'hob' is used. A hob is a cutter shaped like a worm, with its sides gashed similarly to a reamer or tap to form cutting faces. Fig. 119 illustrates the principle, but instead of a hob, an ordinary worm is shown. In a section of the worm along its axis the teeth have straight sides, the same as involute rack teeth, the rack being shown in dotted outline.

The teeth of the worm or hob, being of helical form, like a screw thread, its axis, in order to cut teeth on the blank parallel with the axis of the blank, cannot be perpendicular to the blank, but must be at an angle of 90° minus the helix angle of the worm.

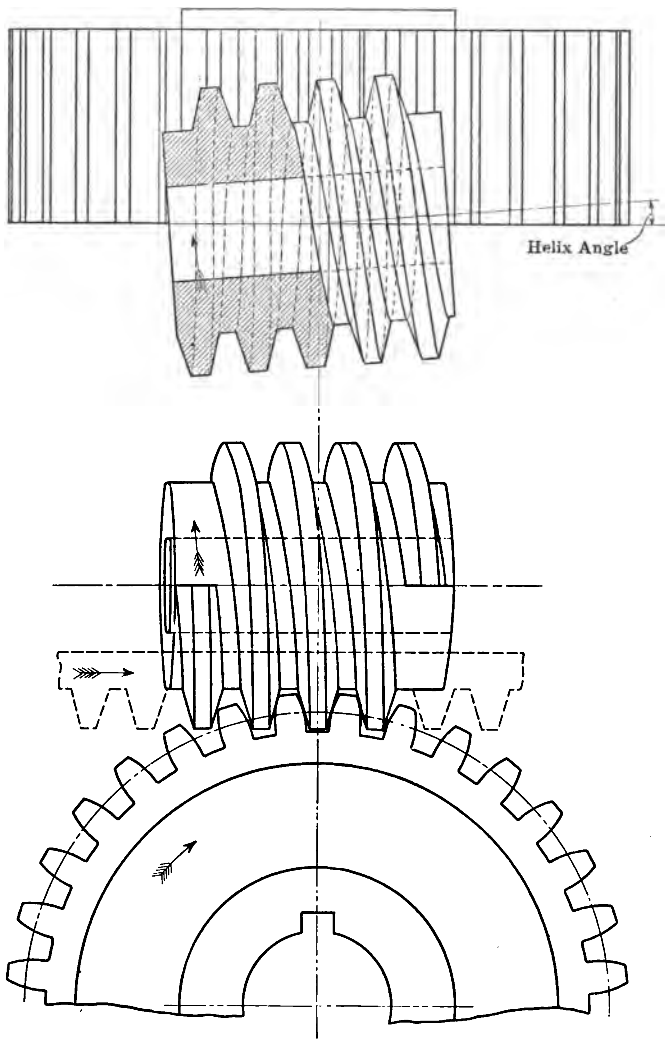


FIG. 119.

Fig. 120 shows a gear hobbing attachment on a universal milling machine, in which the hob is placed on the arbor of the machine with the hob and blank driven at the desired velocity ratio.

The hob is slowly fed across the face of the blank, and when it has gone all the way across the work is completed.

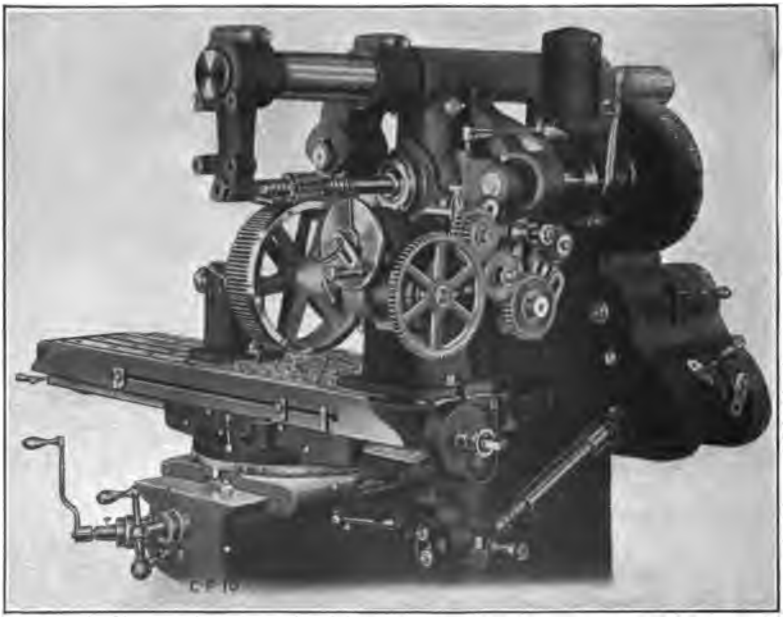


FIG. 120.

CHAPTER VIII

BEVEL GEARS, WORM AND WORM WHEEL

120. In all of the gears thus far considered, the elements of the teeth were parallel to each other, and with the exception oi the helical gears, the elements of the teeth were parallel to the axis of the gear, the tooth outlines being generated by a right line which was the element of a flexible band as in the involute system, or the element of a rolling cylinder in cycloidal system.

In bevel gears, the axes intersect, the pitch cylinders becoming pitch cones, and the elements of the teeth converging at the point of intersection of the shafts.

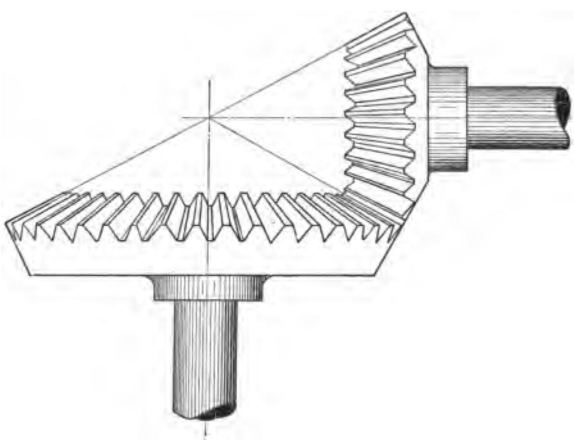


FIG. 121.

An idea of this can be had if it is imagined that alternate grooves and projections are placed on the friction cones of Fig. 83 as was done in Figs. 86 and 87. The cones will then look as in Fig. 121, which shows a pair of bevel gears.

The pitch cones are segments of a sphere, and in order to lay out the gears theoretically correct, the tooth outlines need to be laid out on the surface of the sphere, making use of spherical trigonometry. With the exception of this difference, the bevel gear differs little in theory from the ordinary spur gear.

Bevel gears are always laid out in pairs, and are not interchangeable as are gears for parallel axes.

121. Velocity Ratio.—The velocity ratio for bevel gears is inversely as the radii of the bases of their pitch cones, and the method of laying out a pair of cones for any desired velocity ratio was discussed in Art. 84.

122. Miter Gears.—These are bevel gears that are of the same size and in which the shaft angle is 90°. The velocity ratio in a pair of miter gears is therefore as 1:1.

123. Crown Gears.—A crown gear is one in which the sides of the pitch cone make an angle of 180° with each other. The base of the pitch cone is therefore a great circle of the sphere on which the teeth of the crown gear is developed. The crown gear is sometimes called the rack of bevel gears.

The involute crown gear has not, however, teeth with straight sides as in the involute rack of spur gears. The crown gear with straight sided teeth is known as the *octoid* tooth, so called from the peculiar figure eight line of action. This tooth is the invention of Mr. Hugo Bilgram of Philadelphia, and will be referred to again under bevel gear cutting.

Laying Out the Teeth of Bevel Gears

124. Tredgold's Method.—It was stated above that to be laid out theoretically accurate, bevel gears must be laid out on a spherical surface, and as this is not developable it is necessary to use an approximate method, and the one most commonly employed is known as Tredgold's method.

Let OA and OB, Fig. 122, be the axes of two cones COP and POD respectively, their bases being a spherical surface. Normal to the elements of these cones, draw the cones PAC and PBD, the bases of these second or "back" cones coinciding with those of the cones first drawn. If these back cones are cut along one of their elements and rolled out into the plane of the paper, the radius of the upper one, the apex of which is at A, will be AP, and the length of the arc $PE = PC \times \pi$. The lower cone will have a radius BP, and the length of the arc $PF = PD \times \pi$. On the arcs PE and PF can be laid out the tooth curves, the same as for spur gears having pitch radii AP and BP. The pitch must be such that the gears will have a whole number of teeth. After laying out the tooth curves on the developed back cones,

the cones can be rolled back into their original positions and the teeth drawn on the pitch cones.

It will be noticed that that part of the back cones on which the teeth are laid out differs very little from the spherical surface.

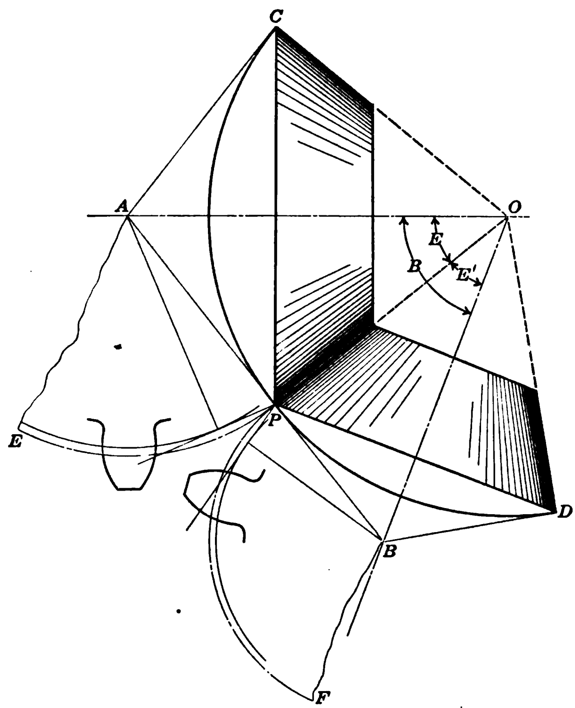


FIG. 122.

Fig. 123 shows one of these cones laid out with half of the teeth drawn in. The teeth in this figure appear in their true thickness in the plan view, but are foreshortened.

The method of construction is clearly shown in the figure so that there is no need of further explanation.

125. Shop Drawing of Bevel Gears.—In making a drawing for use in the shop, it is not necessary to go into the detail of drawing in the tooth curves as in the previous article, but a much more simple drawing together with the necessary angles and dimensions is all that is required.

Problem.—Make a complete working drawing of a pair of bevel gears of 26 and 20 teeth; diametral pitch 4; involute system;

shaft angle 90°, width of face $1\frac{1}{2}$ in.; diameter of shafts 1 in.; diameter of hubs $2\frac{1}{4}$ in.

Make axis of large gear horizontal.

26 teeth, 4 pitch $= 6\frac{1}{2}$ in. pitch diameter.

20 teeth, 4 pitch $= 5$ in. pitch diameter.

Draw the axes of the gears OA and OB, Fig. 124, making an angle of 90° with each other. From O on OA lay off $2\frac{1}{2}$ in., the pitch radius of the small gear and draw CP perpendicular to OA.

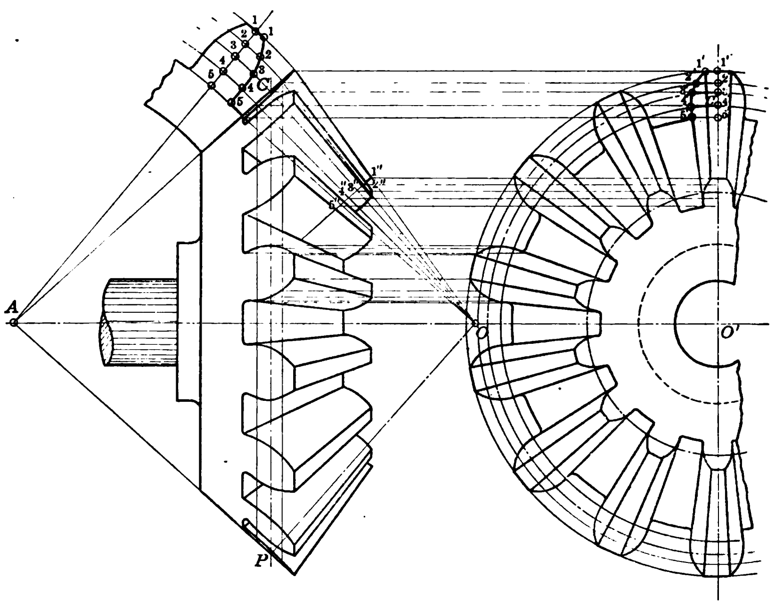


Fig. 123.

On OB from O lay off $3\frac{1}{4}$ in. the pitch radius of the large gear and draw PD perpendicular to OB. Complete the pitch cones POC and POD and draw the back cones, or at least that part of them where the teeth are to be laid out.

The angles E and E' are called the pitch, or center angles.

From the tables of tooth parts find the addendum and dedendum of the teeth and lay them out from C, P and D as shown. From the points thus found, draw toward the apex of the pitch cones and lay off the face of the gears.

The angles that the addendum and dedendum lines make

with the axes of the gears, give the face and cutting angles *F* and *C* respectively.

The angle of the edge, is *T*, and is the same as the center or pitch angle.

The angles *U* and *V* in the small figure are called, respectively, the angle increment and angle decrement. These are the differ-

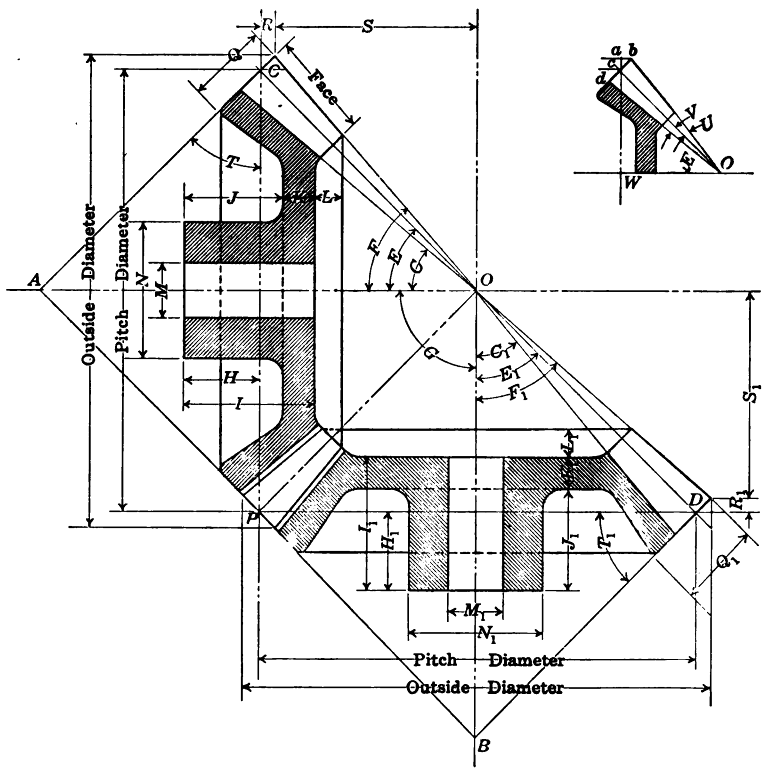


Fig. 124.

ences between the pitch angle and the face and cutting angles respectively.

When the axes of the gears are at right angles, the various angles and the outside diameter can be found as follows:

In the small figure,

Let Wc = pitch radius of small gear

Let OW = pitch radius of large gear

$$\text{Then Tan } E = \frac{Wc}{OW} = \frac{\text{pitch radius of small gear}}{\text{pitch radius of large gear}} = \frac{\text{number of teeth in small gear}}{\text{number of teeth in large gear}}$$

$$OC = \frac{Wc}{\sin E}$$

$$\text{Then Tan } U = \frac{cb}{Oc} \text{ and Tan } V = \frac{cd}{Oc}$$

cb and cd are the face and flank of the tooth, the dimensions being found in the table of tooth parts.

The angle acb = angle E.

Then $ac = cb \cos acb$

$\therefore$ Pitch diameter of large gear $+ 2ac =$ outside diameter of large gear.

In a similar manner the distance R can be found.

It is of course necessary to find these angles and dimensions for each of the gears.

When the shaft angle is not 90° the method is not quite as simple, but after finding the pitch angles, the others are found in a manner similar to that described. By some authorities, R is called the "backing," while others use H as the backing.

The other dimensions necessary for a complete shop drawing are indicated in the figure.

Cutting Bevel-gear Teeth

126. Approximate Methods.—Bevel-gear teeth being on the surface of a cone, their section is different at all points along the elements of the face. It is not possible therefore to cut the teeth theoretically correct with circular cutters. For narrow faces it can be done approximately by using a cutter the shape of the tooth at the large end, but about 0.005 in. less than the thickness of the space at the small end, measured on the pitch line. The settings of the milling table and blank are so made that one side of all the teeth is finished first, then with another setting of the table, the other side is milled.

Sometimes for gears that are to be cut with circular cutters, the blank is turned up so that the addendum line of the face of the gear is parallel with the pitch line, or the face of the cone instead of being drawn to the apex of the pitch cone is drawn

parallel with the pitch cone. In cutting the teeth on such a blank their depth is made constant.

Mr. C. H. Logue says,[1] "The milling of bevel gears is now practically a thing of the past, except for an occasional job required for shop use, the generating of bevel gears having reached such a point that the milling machine cannot compete either in quality or time."

127. Accurately Cut Bevel-gear Teeth.—Fig. 125 illustrates diagramatically a method of cutting the teeth of bevel gears in which a former or templet is used.

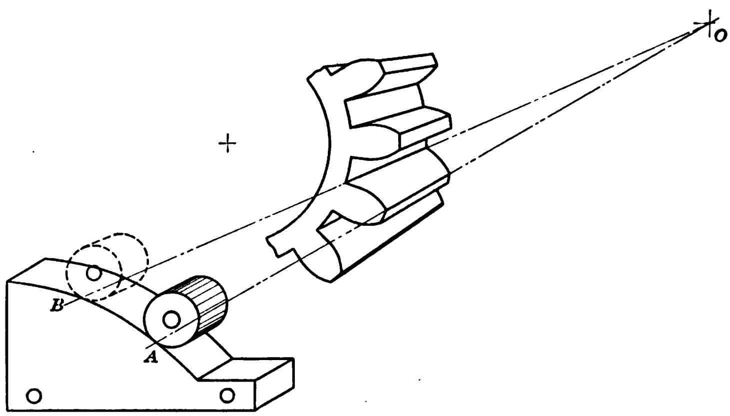


Fig. 125.

The templet is made the shape of the teeth that are to be cut, and being farther from the apex of the pitch cone, is made to a larger scale than the tooth outline. The cutting tool is carried on a frame, the back end of which is supported by a roll resting on the templet.

The cutting point of the tool moves with a reciprocating motion along the line OA, which is tangent to the templet and roll, and passes through the apex of the pitch cone.

Another method of guiding the rear end of the cutting tool frame is to move it in a circular arc by means of a link, instead of by using a templet. This is called the odontograph method.

128. Bilgram Bevel-gear Planer.—This machine bears the same relation to the bevel gear that the Fellows gear planer does to the spur gear, in that the teeth are generated. In Art. 117

[1] American Machinist Gear Book, page 153.

was discussed a method of making interchangeable gears by means of a rack tooth, and all gears that would mesh correctly with the rack would also mesh correctly with each other. In the Bilgram bevel-gear planer, the cutting tool corresponds to the straight-sided tooth of an octoid crown gear.

The space between the teeth of bevel gears being of varying width, however, it is not possible to use a tool the shape of Fig. 116, but only the right or left half of it.

Fig. 126.

The planer illustrated in Fig. 126 is automatic, and consists of two principal parts, the shaper which holds and operates the tool, and the evolver which holds and moves the blank.

The blank must move as a rolling cone, which is done first by securing its arbor in an inclined position to a semi-circular horizontal plate between two uprights. This semi-circular plate can be oscillated on a vertical axis passing through the apex of

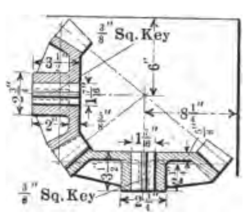
the blank, and is driven by a worm and worm-wheel combination. The second part of the motion is obtained by means of two steel bands stretched in opposite directions, one end of each being fastened to the frame and the other ends wrapped around a cone, which corresponds to the pitch cone of the blank.

The tool which is operated by a Whitworth quick-return motion, cuts on the forward stroke, and during each return stroke the tool is raised clear of the blank by means of a cam, and the blank rotated one tooth by means of gearing and index mechanism. All spaces are treated in the same manner, after which the tool automatically adjusts itself for the second cut on a tooth, and so on until the blank is completed. A similar tool is used to finish the other sides of the teeth.

The inclination of the arbor that holds the blank is made adjustable in order to adapt it to the angle of the desired gear. This adjustment must be exactly concentric with the apex of the blank. Ordinarily a different sized rolling cone would have to be used to cut each different sized gear, but means have been devised whereby a limited number of cones are sufficient.[1]

PROBLEMS

59. Make a finished pencil or ink shop drawing of a pair of bevel gears of 36 teeth and 27 teeth respectively, 3 pitch and 15° angle of obliquity. Face of gears $2\frac{1}{4}$ in.



Note.—Put on all of dimensions and angles shown in Fig. 124. No statement of problem to go on sheet, but this data. BEVEL GEARS

Teeth 36 and 27
Pitch 3 diametral
Involute System
Time, 4 hours

60. Make a sketch showing how you would find the pitch cones for a pair of bevel gears in which the driver made five revolutions to seven of the driven, with a 115° shaft angle.

[1] For a more complete description of this machine see *American Machinist* of May 9, 1885, and Jan. 23, 1902.

Worm and Worm Wheel

129. When two shafts are at right angles but do not intersect, motion can be transmitted between them by means of a worm and worm-wheel combination, which is illustrated in Fig. 127.

Velocity Ratio.—Worm gearing differs from spur gearing in that the velocity ratio does not depend upon the diameters of the gears, but is the relation between the number of threads on the worm (single or multiple) and the number of teeth in the worm wheel. The worm is always the driver, and for a single

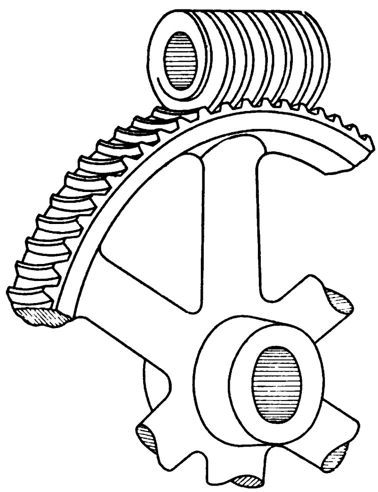


Fig. 127.

thread worm, each revolution of the worm would rotate the worm wheel one tooth, while if it were a double thread worm, the worm wheel would be rotated two teeth and so on. For example, with a single thread worm and a 48-tooth wheel, it would require 48 revolutions of the worm to rotate the where once. If the worm were of double thread, it would require 24 revolutions of the worm for one of the wheel, while if a triple thread worm were used 16 revolutions of the worm would be necessary for one of the wheel.

Thus it will be seen that the pitch diameters do not enter into the question.

If the pitch and number of teeth of the wheel are given, and also the distance between the shaft centers, the pitch diameter of the worm should be such that the two pitch circles are tangent.

130. Pitch.—The circular pitch system is used in making worm and wheel calculations, for the reason that the worms are usually cut in the lathe the same as screw threads, and lathes are not equipped with the proper change gears for cutting diametral pitches.

The objection to the circular pitch system to spur gears, that the distance between the shaft centers will be an inconvenient fraction unless the circular pitch is inconvenient, does not apply in this case however.

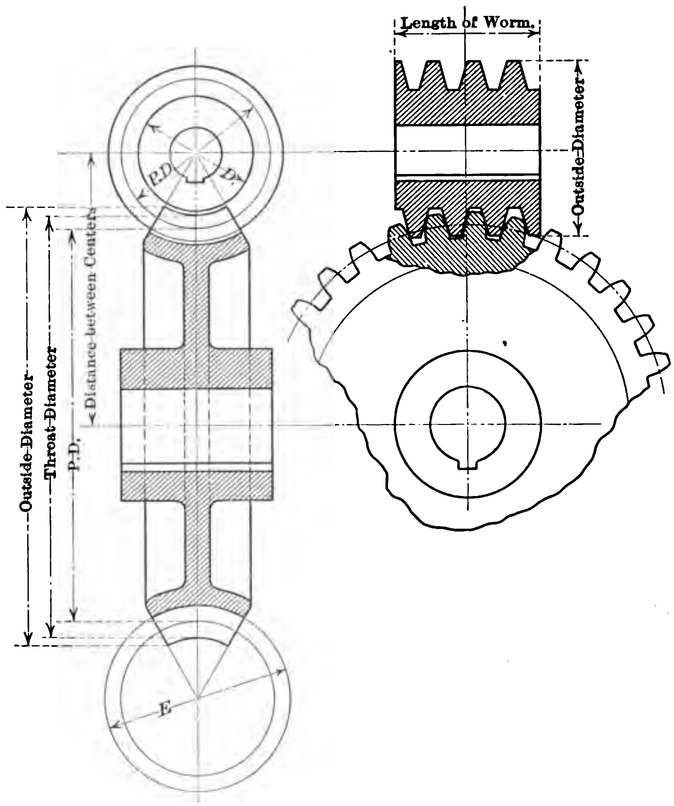


Fig. 128.

The involute system is used in worm gearing for the reason that the worm will have teeth of straight sides which are easily cut in the lathe.

Fig. 128 shows two views of a worm and wheel, the left-hand view being a section along the axis of the wheel and the right-

hand view a section along the axis of the worm. From these views it can be seen that the section of the worm in the right-hand view is the same as an involute rack and the section of the wheel (not wholly cross-hatched), in the same view is the same as that of a spur gear having the same pitch diameter and number of teeth as the wheel. The *throat diameter* of the wheel is the same as the outside diameter of a spur gear having the same pitch and number of teeth.

131. Contact between the worm and wheel teeth is not very definitely determined but it is generally believed to be nearly line contact between any pair of teeth for small gears, although it may have some width, and becomes surface contact for the larger gears. To get as much contact as possible, the worm wheel is constructed so that its sides partially envelop the worm. The included angle between the sides is generally made 60 degrees or 90 degrees.

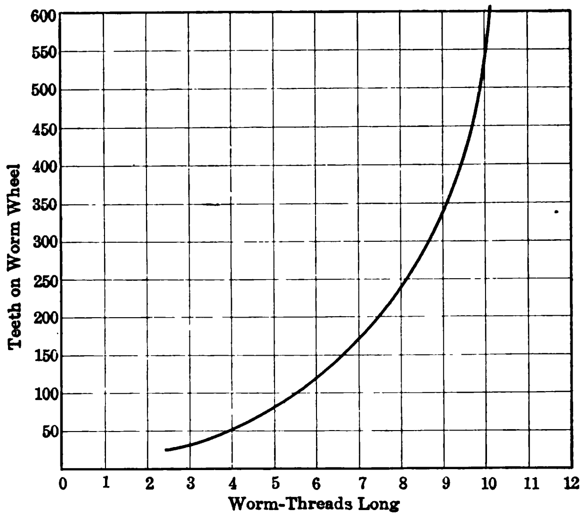


FIG. 129.

132. Length of Worm.—The worm is not limited in length but should be long enough to provide all of the contact that can be obtained between any pair of teeth. It is often made longer than necessary and moved along the shaft when it becomes worn, to bring a new part of the worm in contact with the wheel teeth.

The curve[1] shown in Fig. 129 gives the length of worm required to get the maximum contact when $14\frac{1}{2}$-degree angle of obliquity

[1] From data of Brown and Sharpe Manufacturing Company.

is used. Contact will not be along the whole length of the worm on its axial section, but on the sides as the wheel partially envelops the worm.

133. Cutting the Worm-wheel Teeth.—A worm is first made in the lathe, the same as the worm that is to mesh with the wheel, except that its diameter is twice the clearance greater, then its sides are fluted similar to a reamer or tap in order to form cutting edges. This *hob* and the wheel blank are then mounted in a milling machine, the hob being fed down into the blank until their pitch lines are tangent, then both are turned at the proper velocity ratio. The only difference between hobbing the worm wheel and the spur gear shown in Fig. 120 is that in the worm wheel the axis of the hob is directly over the center line of the blank, and does not move across it.

For worm wheels of less than 30 teeth the flanks will be undercut unless the blank is made over-size.

If the axis of the worm deviates from a right angle with that of the wheel by an amount equal to the thread angle of the worm, the worm can be used with an ordinary spur gear. This is sometimes done in rough work.

Elliptical Gears

134. Elliptical gears are the most common of the non-circular gears.

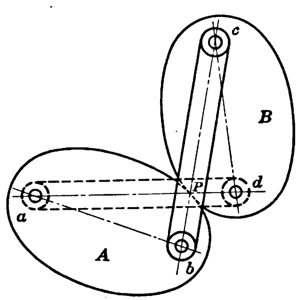


Fig. 130.

In order that two ellipses roll together properly, they must be equal and similar, and each ellipse must rotate about one of its foci, the distance between the centers of rotation being equal to the major axis.

Let A and B, Fig. 130, be two ellipses with foci ab and cd

respectively. The links joining opposite foci form a crossed link kinematic chain similar to Fig. 15 in which the crossed and opposite links are equal. The point of intersection of the crossed links is at the tangent point P, of the ellipses. If one of the ellipses is held stationary, the other can be rolled with pure rolling contact around it.

Velocity Ratio.—Since the two ellipses are the same size, the velocity ratio will be as 1 : 1, but at any part of the revolution the ratio is inversely as the radii from the fixed foci to the tangent point P, of the ellipses.

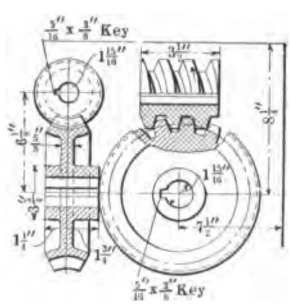
Any system of teeth that is practicable for spur gears can be used for elliptical gears and the teeth if cut with the same cutter will all have different profiles. This may result in weak under-cut teeth at and near the ends of the ellipses, but can be remedied by using one cutter for the ends where the ellipse has a small radius, and another for the sides where the radius is larger. The blanks are usually cut by clamping them together and cutting both at one operation. This insures the mating teeth being the same.

Elliptical gears can be used for "quick-return motions" such as slotters and shapers, where all of the work is done on half of the revolution, the other half being used to get the tool out of the way and the work ready for the next stroke.

They are not in very general use as they are hard to cut without the use of special attachments.

PROBLEMS

61. Make a full-size working drawing of a worm and worm-wheel combination. Involute system.



Data: Velocity ratio 32 : 1.
Single thread right-hand worm.
Circular pitch $\frac{7}{8}$ in.

Note.—Make teeth in contact at pitch point. Draw tooth outlines of wheel teeth by means of Grant's Involute Odontograph table. Develop helix for outline of worm teeth. No statement of problem, but this data,

Worm & Worm Wheel.
Teeth in wheel 32
Single thread R.H. worm
Circular Pitch $\frac{7}{8}$ in.
Involute System
Time, 4 hours.

CHAPTER IX

GEAR TRAINS

135. When more than two gears are in mesh, such a combination is a train of mechanism (Art. 16), and is called a *gear train* or train of gears.

It is often desirable to find the revolutions of the last gear of a train without finding the revolutions of each intermediate gear.

The number of teeth on any two gears that mesh together is proportional to their respective pitch diameters, and since the revolutions are inversely proportional to the pitch diameters (Art. 15), therefore, their revolutions are also inversely proportional to the number of teeth on the gears.

136. Value of a Train.—By this expression is meant the ratio of the revolutions of the first and last gear of the train in a given time.

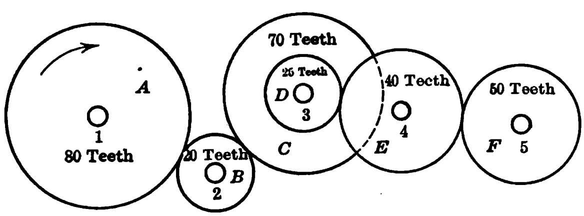


FIG. 131.

In Fig. 131 let A, B, C, D, E, and F be the pitch circles of gears on the shafts 1, 2, 3, 4 and 5, and having the number of teeth as shown.

The gear A drives B, B drives C, and since C and D are on the same shaft they will rotate as one gear. D drives E and E drives F.

$$\frac{\text{Revolutions of } B}{\text{Revolutions of } A}=\frac{80}{20}; \quad \frac{\text{Revolutions of } C}{\text{Revolutions of } B}=\frac{20}{70}$$

$$\frac{\text{Revolutions of } E}{\text{Revolutions of } D}=\frac{25}{40}; \quad \frac{\text{Revolutions of } F}{\text{Revolutions of } E}=\frac{40}{50}$$

$$\text{Or, } \frac{\text{Revolutions of } F}{\text{Revolutions of } A}=\frac{80}{20}\times\frac{20}{70}\times\frac{25}{40}\times\frac{40}{50}=\frac{4}{7}$$

That is for each revolution of A, F will make $\frac{4}{7}$ of a revolution. Instead of the number of teeth, the pitch diameters can be used. A rule for the above may be expressed as follows: To find the number of revolutions of the last gear of a train, divide the continued products of the teeth or pitch diameters of all the drivers times the revolutions of the first gear, by the continued products of the teeth or pitch diameters of all the driven gears.

It is not necessary that the same unit be used throughout the various ratios, but the same unit must be used in the same ratio. That is, we may have a 60-tooth gear driving a 45-tooth gear and a 12-in. gear driving a 6-in. gear, and so on, but we cannot use a 60-tooth gear driving a 12-in. gear.

A gear train may be written in the following manner to avoid making a sketch of the gears.

First axis,	64 teeth.				
Second axis,	32 –	10″			
Third axis,		15 –	50 teeth.		
Fourth axis,			20		
Fifth axis,			30 –	12″ (annular).	
Sixth axis,				3 –	24 tooth.

All of the gears on the same horizontal line are on the same shaft, and each gear drives the one directly below it.

If the first gear of this train makes 60 revolutions per minute, the value of the train will be $\frac{64\times10\times50\times20\times12\times60}{32\times15\times20\times30\times3}=53\frac{1}{3}$ revolutions per minute for the sixth axis.

137. Idle Gear and Direction of Rotation.—An idle gear is a gear placed between two axes for the purpose of changing the directional relation of the two. Thus if two gears are placed in direct contact their direction of rotation will be opposite, but if one intermediate gear is placed between them their direction of rotation will be the same. The idler does not change the velocity ratio, since it is both a driver and a driven. In Fig. 131, the gears B and E are idlers and the velocity ratio would not be affected if they were left out. The gears C and D however are not idlers although they are on the same shaft, since the number of teeth on them is not the same.

In spur gears when there is an odd number of intermediate axes between the first and last, the direction of rotation of the

first and last axes will be the same, while if there is an even number of intermediate axes, the direction of rotation will be opposite.

If there is one annular gear in the train, it changes the direction of rotation of the last gear from what it would be otherwise. Thus in the last problem worked out there are four intermediate axes and one annular gear, so that the direction of rotation of the first and last axes will be the same.

138. Gear Train for Thread Cutting.—Fig. 132 illustrates the gearing found on an ordinary engine lathe that is equipped for cutting screw threads. On the spindle *E*, are the gears *F*, *C* and

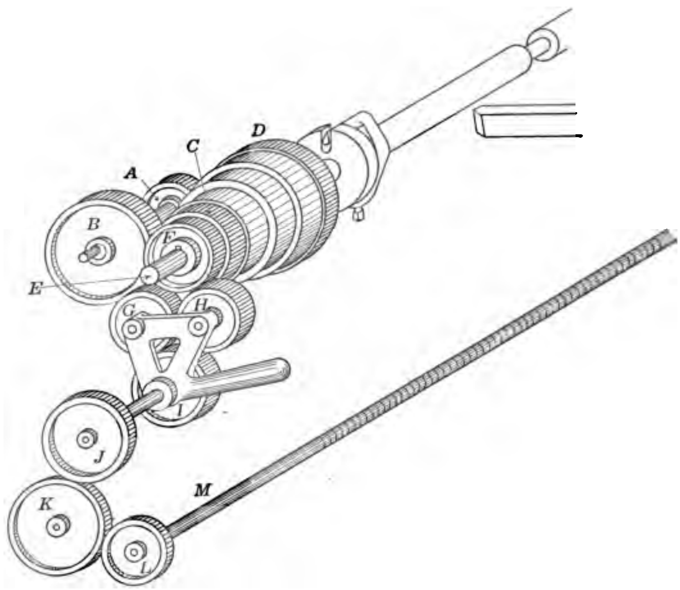


FIG. 132.

D and the stepped cone. The gear *D* is keyed to the spindle and the stepped cone is free to revolve on the spindle except when it is locked to *D* by means of a pin.

The gear *C* is fastened to the stepped cone and revolves with it. *A* and *B* are the *back gears* and are keyed to a shaft that is parallel to the spindle. This shaft has eccentric bearings so that the back gears may be thrown out of mesh with *C* and *D*.

When the back gears are "thrown out," *D* and the stepped cone are locked together, and as many speeds can be obtained as there are steps on the cone, while with the back gears "thrown

in," D and the stepped cone are not locked and the number of speeds of the spindle is doubled.

F is the spindle gear and is for driving the thread-cutting train. G and H are tumbling gears, or tumblers and are for the purpose of changing the direction of rotation of the screw M.

G meshes with I which is the inside stud gear, and the arrangement is such that F and H may be thrown out of mesh and G be made to mesh directly with F. F and I are generally made the same size, but when they are not the same, I is usually made twice the size of F, so that I makes one-half as many revolutions as the spindle.

On the same shaft with I is the outside stud gear J. Meshing with J is an intermediate K which meshes with the screw gear L on the end of the lead screw M. The only gears that it is necessary to change in thread cutting are the stud gear J and the screw gear L. The intermediate K is held on a slotted bracket, not shown in the figure, which allows K to be adjusted to accommodate different sized gears J and L.

The cutting tool is held in a tool holder which is moved back and forth along the work by means of a split nut engaging the lead screw.

If there are eight threads per inch on the lead screw, then eight revolutions of the lead screw would advance the cutting tool one in., and if the work made four revolutions to eight of the lead screw, four threads per inch would be cut on the work.

To find the number of teeth in the stud and screw gears when F and I are of the same size. Since the gears G, H and K are idlers they need not enter into the calculations, and since the spindle and stud shafts make the same number of revolutions it is only necessary to find the revolutions of the stud shaft.

Let $N=$ number of threads per inch to be cut on work.
Let $n=$ number of threads per inch on lead screw.
Let $T=$ number of teeth on screw gear.
Let $t=$ number of teeth on stud gear.

$$\frac{\text{Revolutions of stud shaft}}{\text{Revolutions of lead screw}}=\frac{\text{Number of teeth on screw gear}}{\text{Number of teeth on stud gear}}$$

or $\frac{N}{n}=\frac{T}{t}$ and $T=\frac{N}{n}\times t$.

The following table of change gears was taken from the plate on a 16 in. swing lathe, having six threads per inch on the lead

screw and twice as many teeth on the inside stud gear as on the spindle gear.

Threads to be cut	Teeth on stud gear	Teeth on screw gear	Threads to be cut	Teeth on stud gear	Teeth on screw gear
3	80	20	15	48	60
4	72	24	16	48	64
5	48	20	18	48	72
6	48	24	20	24	40
7	48	28	22	24	44
8	48	32	24	24	48
9	48	36	26	24	52
10	48	40	28	24	56
11	48	44	30	24	60
11½	48	46	32	24	64
12	48	48	36	24	72
13	48	52	40	24	80
14	48	56	48	20	80

To obtain a greater range of threads without adding more change gears, the intermediate gear K, on Fig. 132, might be *compounded,* that is, replaced by two gears of different sizes, and fastened together, one of them meshing with the stud gear and the other with the screw gear.

The involute system is used for change gear sets for in this system the center distance can be varied without affecting the velocity ratio.

PROBLEMS

62. How many r.p.m. does the last gear of the following train make, if the first gear makes 25 r.p.m.? Is its direction of rotation the same or opposite?

12 in.
6 in.
20 in. $-$6 in.
12 in.
20 in. $-48T$ (annular)
$24T-12$ in.

63. The first gear of the following train makes 100 r.p.m. and turns clockwise. How many r.p.m. does the last gear make?

$60T-4P$.
15 in.
10 in. $-$12 in.
15 in.————$30T$ (bevel)
$36T$ (bevel)—single Thd worm
$120T$ worm wheel—10 in.

64. Design a lathe screw train for right- and left-hand thread cutting, to cut 8, 9, 10, 11, 11½, 12, 13, 14, 16, 18, 20, 24, and 27 threads per inch.

Data: Threads per inch on lead screw 8.
Teeth in spindle and inside stud gears 24.
Teeth in tumblers 21.
Teeth in intermediate 108.
All gears 12 pitch.

Note.—Make a full size pencil drawing, representing the extreme positions of the gears by their pitch circles.

CHAPTER X

BELTING

139. Belts are flexible connectors and are used for connecting shafts where the distance is too great for gears, or where an absolutely constant velocity ratio is not required.

Belting may be divided into two general classes, flat and round; the former is used on pulleys with faces that are cylindrical or nearly so, and the latter which is usually either of fiber or metal requiring pulleys with grooves or flanges to keep the rope from running off the pulley or drum.

140. Velocity Ratio and Directional Rotation.—When two pulleys are connected by an inelastic belt of no appreciable thickness, all parts of the belt and the surfaces of the pulleys have the same linear velocity, if there be no slipping and therefore their revolutions are inversely proportional to their radii.

This condition is not attained in practice on account of the elasticity and thickness of the belt. In order to be accurate the thickness of the belt should be added to the diameters of the pulleys in making calculations. In this discussion, where the diameter of the pulley is referred to, it means diameter of pulley plus thickness of belt.

When the velocity ratio between two shafts is large, instead of making a single reduction, it may be made in one or more steps by using a *counter shaft* or *jack shaft*. This is an intermediate shaft on which there are two pulleys of different diameters. Thus if the velocity ratio between two shafts were 1 : 40, the ratio might be divided up 1 : 5 and 1 : 8.

The method of connection shown in Fig. 133 is an *open* belt, while that shown in Fig. 134 is a *crossed* belt.

With an open belt both pulleys turn in the same direction, or the directional relation is the same, while in the case of the crossed belt, the directional relation is opposite.

In order that a belt may maintain its position on a pulley, its *center line on the approaching*[1] *side must lie in a plane perpen-*

[1] By the approaching side is meant the part of the belt advancing toward the pulley. The side where the belt leaves the pulley is called the receding side.

dicular to the axis of the pulley. If a force is applied to the approaching side of the belt parallel to the axis of the pulley, each succeeding part of the belt will take up a position a little farther along on the face of the pulley. A force applied to the receding side of the belt will have no effect on that pulley unless it is great enough to move the belt bodily. However the receding side with respect to one pulley is the approaching side with respect to the other pulley.

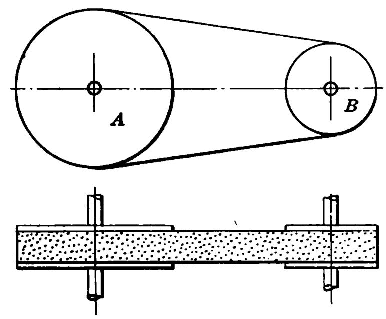


Fig. 133.

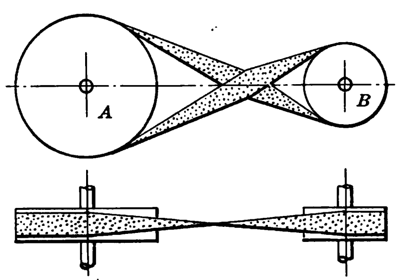


Fig. 134.

141. Crowning Pulleys.—Because of the fact that belts do not remain exactly straight and the pulleys in exact alignment, some means must be provided for keeping the belts from running off, and this can be accomplished by means of flanges on the pulleys, or a fork placed on each side of the belt, but the most common method is to *crown* the pulley. By this is meant that the diameter of the pulley is greater in the center than at the edges. The crown can be either straight or spherical and its amount will depend upon the width and speed of the belt, and the distance between the centers of the pulleys. Less crown is required the

wider the face of the pulley, the greater the speed and the greater the distance between the pulley centers.

142. Tight and Loose Pulleys.—When machines are driven from a line shaft, and it is not desirable to have them run all of the time, means must be provided so that any machine may be stopped without stopping the line shaft. Common methods of doing this are by means of clutches, or the tight and loose pulley combination. The tight and loose pulleys are two pulleys placed side by side on the counter shaft, the tight pulley being keyed to the shaft, and revolving with it, while the loose pulley is held in place by a collar, and is free to revolve on the shaft. When the machine is thrown out of operation the belt is shifted from the tight pulley to the loose pulley. The loose pulley should be made of slightly smaller diameter so that the belt will not be stretched as tightly when running idle.

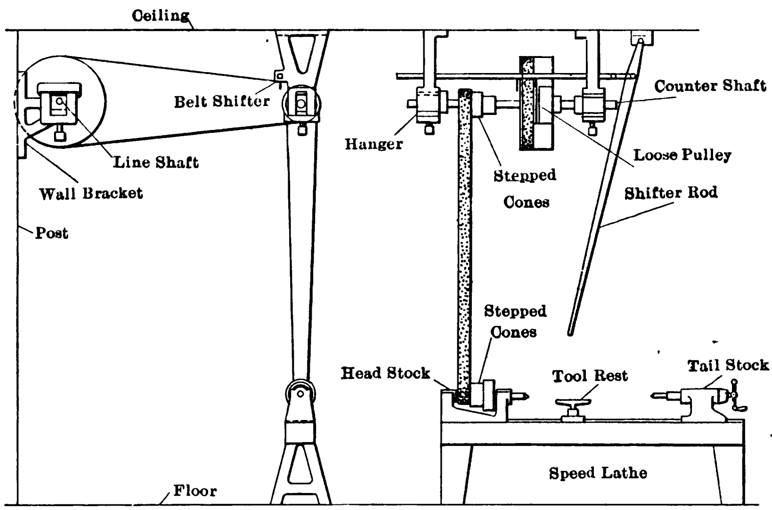


FIG. 135.

Fig. 135 illustrates a method of connecting a lathe with the line shaft by means of belts.

The pulley on the line shaft, over which the belt runs from the tight or loose pulley is made with a cylindrical face, and a width at least equal to that of the tight and loose pulleys combined.

143. In Fig. 136 is shown a method of connecting two shafts with a belt when they are too close together to connect them by the methods of Figs. 133 or 134.

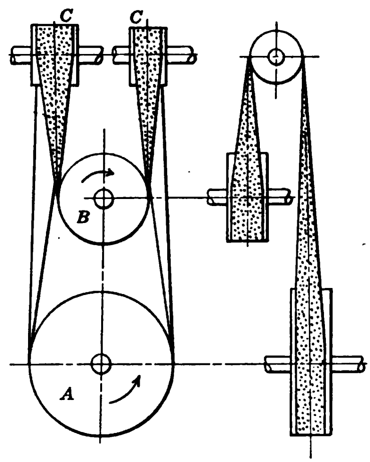


FIG. 136.

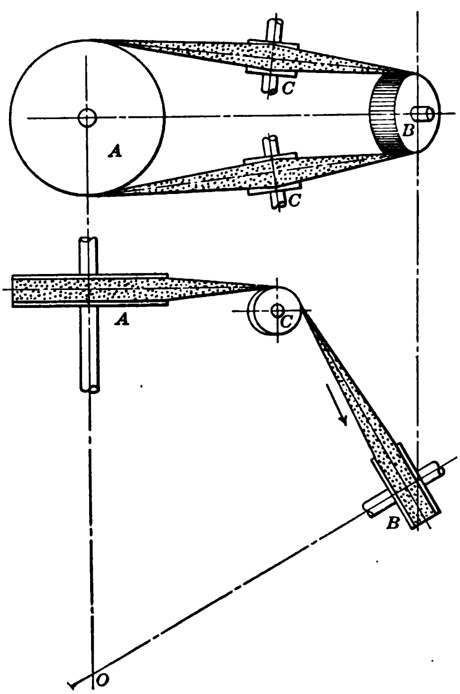


FIG. 137.

144. Belts for Intersecting Shafts.—When two intersecting shafts are to be connected by shafts, guide pulleys must be so placed that the belt will run on the pulley straight (Art. 140). This is accomplished in Fig. 137 by using guide pulleys CC, the axes of which are perpendicular to a tangent to the pulleys A and B shown in the upper view, and their faces are tangent to the center line of the pulleys A and B in the lower view.

145. Belts for Shafts that are neither Parallel nor Intersecting.—The most common form of this arrangement is the "quarter turn" belt in which the shaft angle is 90 degrees.

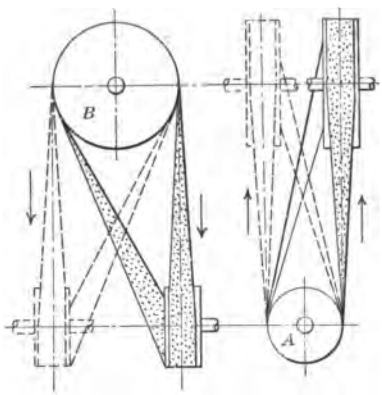


Fig. 138.

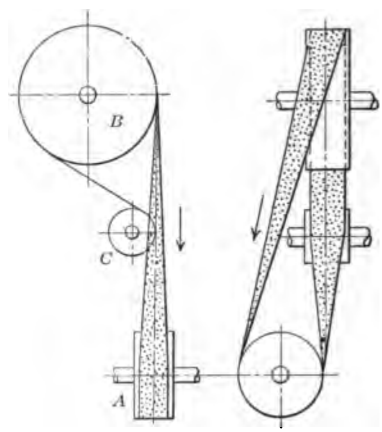


Fig. 139.

Fig. 138 shows two views of such a combination. The main point to bear in mind when adjusting the pulleys for a belt of this kind is to have it run on the pulley straight. This is accomplished if a plane passed through the middle of the face of each pulley, and perpendicular to its axis is tangent to the face of the other pulley. It will be noted that after the pulleys are adjusted, the belt will run in one direction only. In order to have it run in the opposite direction, each pulley must be moved along its shaft equal to the diameter of the other, or else a guide pulley must be used as shown in Fig. 139.

146. Length of Belts.—If the distance between the centers of two shafts and the diameters of the pulleys are known, the length of belt required for them can be calculated as follows:

1. *For Open Belts.* See Fig. 140.

Let L = distance between pulley centers in inches.
Let R = radius of large pulley in inches.
Let r = radius of small pulley in inches.

The total length of belt is the length not in contact with either pulley plus the length in contact with the large pulley plus the length in contact with the small pulley.

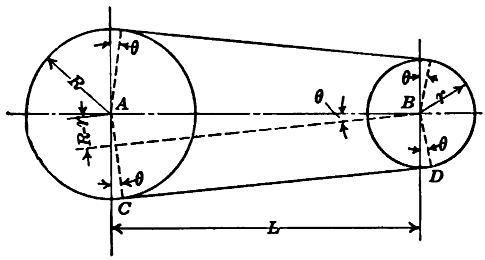


FIG. 140.

Let C and D be the tangent points of the belt and pulleys. Then 2 CD=length of belt not in contact with the pulleys.

$$2\,CD = 2\sqrt{L^2-(R-r)^2}$$

Length of belt in contact with large pulley

$$=R(\pi+2\theta)=R\left(\pi+2\sin^{-1}\frac{R-r}{L}\right)$$

Length of belt in contact with small pulley

$$=r(\pi-2\theta)=r\left(\pi-2\sin^{-1}\frac{R-r}{L}\right)$$

∴Total length of open belt

$$=2\sqrt{L^2-(R-r)^2}+R\left(\pi+2\sin^{-1}\frac{R-r}{L}\right)+r\left(\pi-2\sin^{-1}\frac{R-r}{L}\right)$$

$$=2\sqrt{L^2-(R-r)^2}+\pi(R+r)+2(R-r)\sin^{-1}\frac{R-r}{L}$$

For large distances between shafts and with pulleys of nearly the same diameter, the third part of the above equation may be omitted.

2. *For Crossed Belts.* See Fig. 141.

Using the same notation as before, the tangent points of the belt and the pulleys are C and D, then $2CD=$ the length of belt not in contact with the pulleys.

$$2CD=2\sqrt{L^2-(R+r)^2}$$

Length of belt in contact with large pulley

$$=R(\pi+2\theta)=R\left((\pi+2\sin^{-1}\frac{R+r}{L}\right)$$

Length of belt in contact with small pulley

$$=r(\pi+2\theta)=r\left(\pi+2\ \sin^{-1}\frac{R+r}{L}\right)$$

$\therefore$ Total length of belt

$$=2\sqrt{L^2-(R+r)^2}+R\left(\pi+2\sin^{-1}\frac{R+r}{L}\right)+r\left(\pi+2\sin^{-1}\frac{R+r}{L}\right)$$

$$=2\sqrt{L^2-(R+r)^2}+(R+r)\left(\pi+2\sin^{-1}\frac{R+r}{L}\right)$$

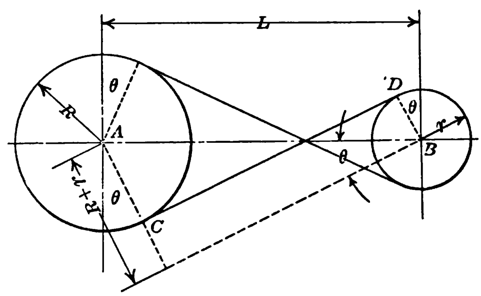


Fig. 141.

It will be noticed that for open belts, the sum and difference of the radii of the pulleys are used, while in the case of crossed belts, only the sum of the radii are considered.

147. Stepped Cones for Same Length of Belt.—On most machine tools the driving pulley is usually stepped, the belt running over a similar one on the counter shaft, and as the same belt must be used on the different pairs of steps they must be designed with this in mind.

1. *For Open Belts.*—No simple formula has been derived that can be used for calculating the size of steps on which an open belt is to be used, so this problem is generally solved graphically.

An approximate method often used is that of C. A. Smith[1] and is as follows:

Let A and B, Fig. 142 be the centers of the shafts, and L the distance between them in inches.

At C half way between A and B, erect a perpendicular CD of length equal to 0.314 L; (this coefficient was determined experimentally[2]).

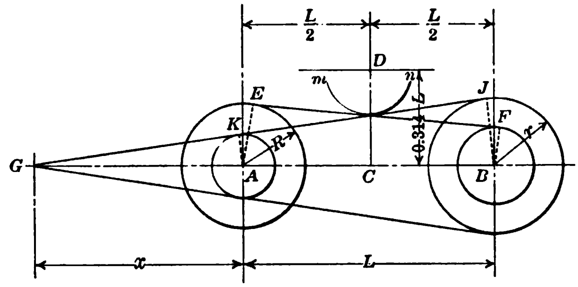


FIG. 142.

Draw a line EF tangent to the pulleys at E and F. With D as a center draw the arc mn of such a radius that it will be tangent to the line EF.

Let BJ be the radius of one of the pulleys of the second pair of steps; to find its mate draw the line JG tangent to the arc mn and tangent to an arc of radius BJ. From A drop a perpendicular AK to this line which will be the radius of the mate.

If instead of the diameter of one of the second pair of steps, the velocity ratio of the second pair together with the diameters of the first pair is given, the solution will be as follows:

Let the distance from where the line JK extended, intersects the line of centers at G, to A equal x. Let the velocity ratio of $\frac{A}{B}=\frac{BJ}{AK}=a$.

Then by similar triangles $\frac{x+L}{x}=\frac{BJ}{AK}$ $\therefore x=\frac{L}{a-1}$

So that by taking $GA=x=\frac{L}{a-1}$ and drawing a line through G tangent to FJ, pulleys for the required velocity ratio will be obtained.

[1] See Transactions American Society of Mechanical Engineers, Vol. X, page 269.

[2] When the angle between the belt and center line of pulley exceeds 18 degrees, CD is taken as 0.298 L.

When the smaller pulley is at B, the value of a becomes less than 1, and that of x negative, which indicates that it must be laid off from A toward the right in the figure.

2. *For Crossed Belts.*—In designing stepped pulleys for a crossed belt, it is only necessary to keep the sum of the radii for the different pairs of steps constant. Thus if the radii of one pair of steps is 18 in. each, their sum is 36 in. and the radii of the other pairs of steps that would take the same length of crossed could be 24 in. and 12 in., 30 in. and 6 in. and other radii in which their sum is 36 in.

Rope Driving

148. In this form of belting, the pulleys must be provided with grooves to guide the rope. For fibrous ropes these grooves are generally of V section, the rope pulling down into them and increasing the tractive power.

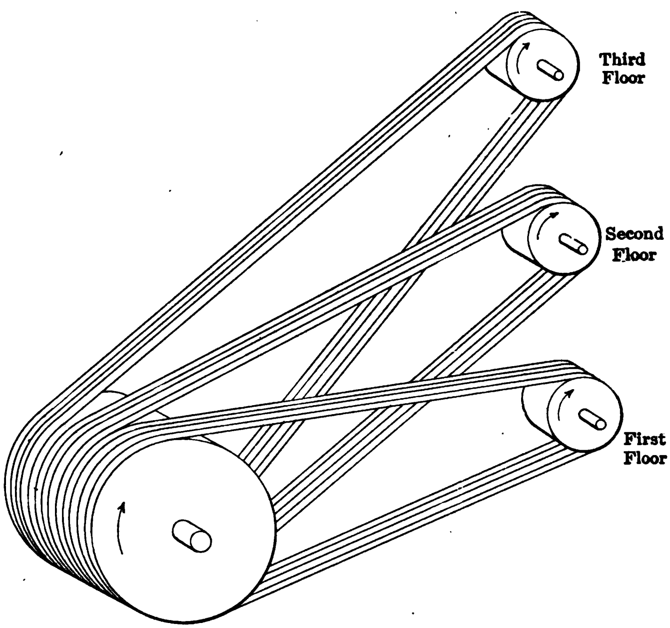


Fig. 143.

For metal rope or cable, this form of groove would be injurious to the cable, so it is made of semi-circular section, the cable resting on the bottom of the groove which may have a leather, gutta-percha or wood insert to prevent metal rubbing on metal.

There are two systems of rope driving in general use, the English or individual rope system, which has a separate rope for each groove and the American or single rope system in which a single rope is carried around all of the grooves and brought back from the last to the first by means of a guide pulley set at an angle with the other pulleys.

These two systems are shown diagrammatically in Figs. 143 and 144. With the individual ropes, a single rope may break and the load be carried by the others, which is not the case with the single rope system. On the other hand, it is difficult to obtain uniform tension on all of the ropes. This is accomplished in the single rope system by means of a tension pulley and weight, so that the wear on the rope is more uniform throughout its length than on the separate ropes of the individual system.

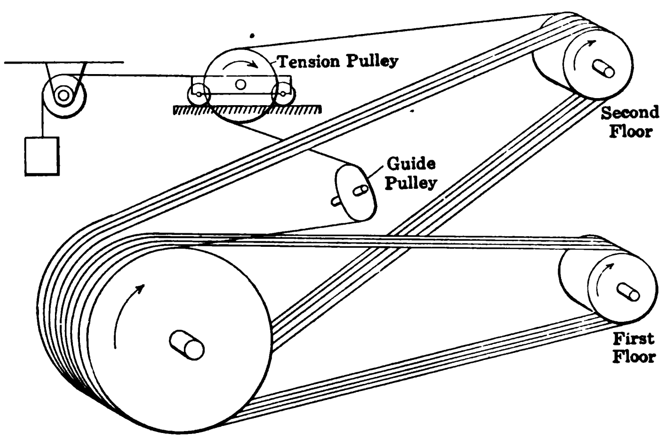


Fig. 144.

Chain Driving

149. The chain drive is used generally where the work is heavy and a positive motion is required at a comparatively low speed. The wheels over which the chains run have teeth to receive alternate links of the chain and are called *sprocket* wheels. The wheels shown in Figs. 145 and 146 have alternate single and double teeth to receive the chain, in other cases the teeth of the wheels are all alike as in bicycle sprocket wheels.

In Fig. 145 the curve of the tooth may be found by taking a

center at A and a radius AB minus the radius of the end of the chain link.

To reduce friction the pins are often provided with rolls as in pin gearing. This form of chain is used in automobile drives.

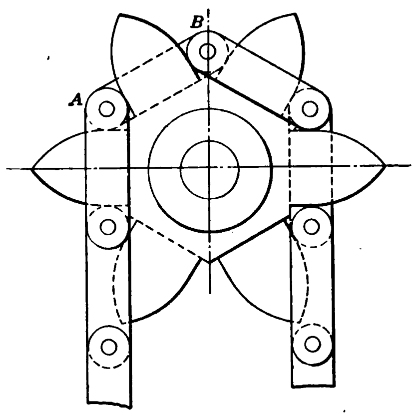


FIG. 145.

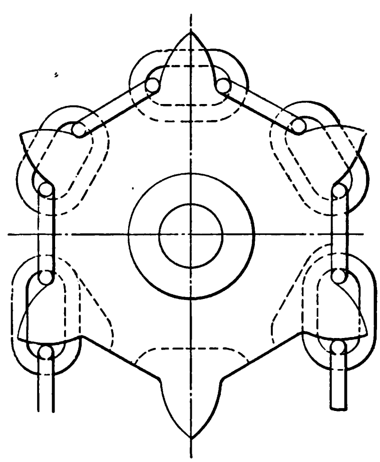


FIG. 146.

To find the pitch diameter of an ordinary sprocket wheel,
Let P', Fig. 147 be the pitch or length of one link of the chain.[1]
Let $T =$ number of teeth in wheel.
Let c = pitch radius of wheel.

[1] This can be readily found by taking a piece of chain, measuring its length and dividing by the number of links.

Let b = one-half of the chordal pitch.

$$\phi = \frac{360°}{2T} \text{ then } \sin\phi = \frac{b}{c}; \text{ and } c = \frac{b}{\sin\phi}$$

After chains have been used for some time, the pitch may change on account of the links stretching and the pins becoming worn, so that if the chain originally fitted the teeth, it will not later and the strain may be all carried on a single tooth.

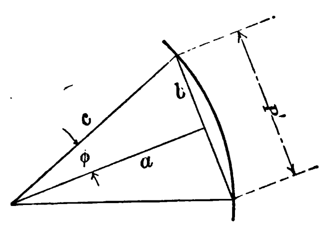


FIG. 147.

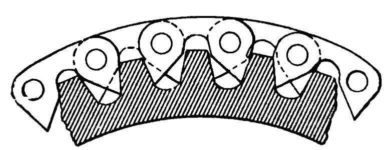


FIG. 148.

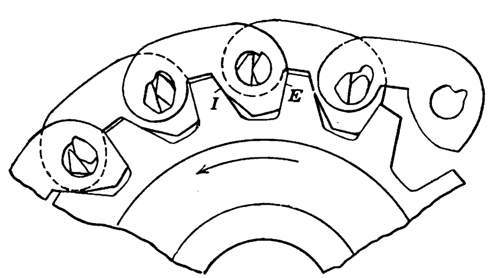


FIG. 149.

150. Reynold's Silent Chain.—This is the invention of Hans Reynolds and is a great improvement in chain drives.

A short section of the chain and sprocket is shown in Fig. 148. The chain is built up of flat pieces of which lap by each other and are fastened together by steel pins. The sides of the teeth that come in contact with the wheel as well as those of the wheel are made straight.

It is very satisfactory for motor drives where a positive motion is desired and the distance between the shafts too small for belting. It also always fits the wheel and takes up the backlash.

151. Morse Rocker Joint Chain.—This chain shown in Fig. 149 like the one just described, is built up of flat pieces of steel but of a different shape. The pins instead of being a single round pin consist of two hardened steel parts so formed that they will rock together and reduce friction. This chain can be used in place of belting for speeds up to 2000 ft. per minute.

PROBLEMS

65. A countershaft makes 50 r.p.m. and has on it a stepped cone, the diameters of the steps being 4 in., 6 in., and 8 in. respectively. The diameters of the corresponding steps on the machine spindle are 9 in., 7 in. and 5 in. respectively. The number of teeth in the back gears are 40 and 25, and the gears that mesh with them have 25 and 35 teeth. How many changes of speed can be obtained, and what will be the r.p.m. of the work for each?

66. Calculate the lengths of crossed and open belt required for two pulleys of 10 ft. and 4 ft. in diameter respectively, and 25 ft. between centers.

67. An emery wheel 8 in. diameter is to have a peripheral speed of 2500 ft. per minute. It is driven by means of a countershaft upon which there are two pulleys, one 18 in. diameter driven by means of a belt from a pulley 36 in. diameter on the main shaft, and the second which drives a pulley 2 in. diameter on the emery-wheel shaft. What must be the diameter of the second pulley on the countershaft if the lineshaft makes 80 r.p.m.?

68. Design a pair of stepped pulleys for a $\left\{ \begin{matrix} \text{crossed} \\ \text{open} \end{matrix} \right\}$ belt in which the distance between the centers is 24 in., diameter of smallest step 4 in., diameter of shafts 1 in. Revolution per minute of driver 200. Revolutions per minute of driven to be 50, 100, 200, 400, and 800, respectively for each of the pairs of steps.

Note.—Lay out the pulleys according to the formulas for crossed belt and by Smith's graphical method for open belts. Check the diameters of each pair of pulleys by calculating the length of belt required. Make the drawing half size. Time, 4 hours.

CHAPTER XI

INTERMITTENT MOTIONS

RATCHET GEARING

152. Ratchet Gearing.—This is perhaps the most common form of intermittent motion and consists of two principal parts, the *ratchet wheel* and *pawl* or *detent.*

A simple form is shown in Fig. 150. The center of the ratchet wheel is at O, and the driving pawl has a center at A on the arm OA.

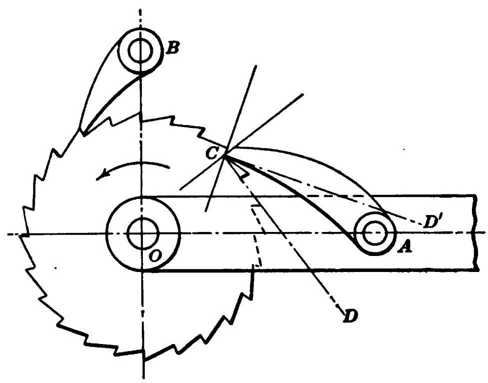


FIG. 150.

The wheel is prevented from turning backward by the pawl with a center at B.

The wheel teeth and pawl should be so shaped that when a load is applied, the pawl will not tend to leave contact with the wheel tooth. To obtain this result a common normal to the front face of the pawl and teeth should pass between the pawl and ratchet wheel centers. CD is such a normal. If the teeth and pawl were so shaped that the common normal were CD', the pawl would then tend to disengage itself from the wheel when under load.

153. In using the pawl with center at B, to prevent backward motion of the wheel, it will only do it after the wheel has rotated

backward far enough for the pawl to engage the teeth, which may vary from zero to the pitch of the teeth.

This backward motion can be reduced by making the pitch smaller, which weakens the teeth, and is not always desirable. It can also be reduced by placing several pawls side by side on the pin and making them of different lengths. In this way the backward motion can be reduced to the pitch divided by the member of pawls.

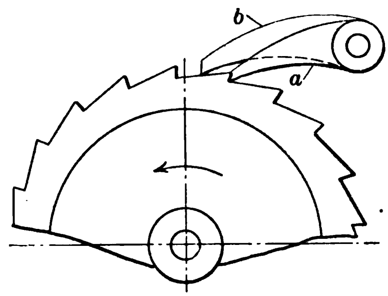


FIG. 151.

By placing a number of driving pawls side by side and having them of different lengths, a very fine feed can be obtained without decreasing the pitch. An illustration of this is the set works on sawmill log carriages. Here a fine feed is required for sawing various thickness but fine teeth would not stand the wear and tear.

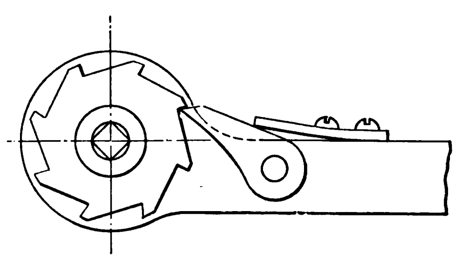


FIG. 152.

Fig. 151 shows this arrangement with two pawls in which one is made one-half of the pitch of the teeth longer than the other.

Fig. 152 illustrates a simple ratchet wheel and pawl as applied to the Weston Ratchet or Scotch drill. The friction of the drill in the hole in this case prevents backward motion. Fig. 153 shows a section of the "Armstrong Universal Ratchet," in which

there are 12 teeth on the wheel and four pawls, which engage one at a time. The universal quality of the tool as claimed by the makers is due only to the fact that the axis of the two trunnions on which the handle turns, is at an acute angle with

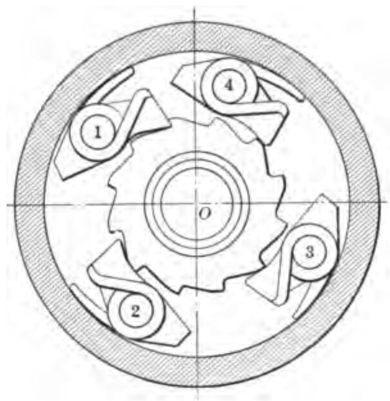
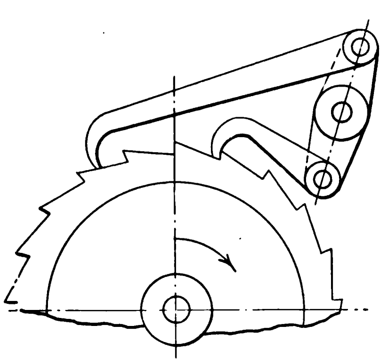


FIG. 153.

the axis of the drill, so that a very small motion of the end of the handle will turn the drill.

To get a motion of the wheel in one direction for both the forward and backward motions of the pawl lever the methods of Figs. 154 or 155 may be employed.



FIGS. 154.

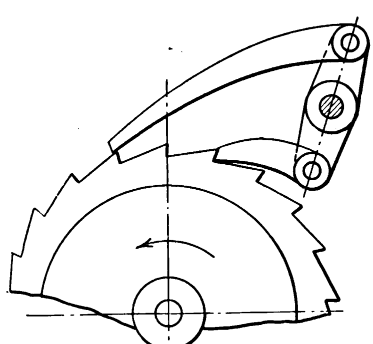


FIG. 155.

If it is desired to get a motion of the wheel in either direction, the form of reversing pawl shown in Fig. 156 may be used. In this case it will be noted that the form of wheel teeth are different

from where motion in one direction only is required. This form is sometimes used on machine tool feed mechanisms.

When it is necessary to vary the motion of the ratchet wheel without varying that of the pawl, it may be done by "masking" one or more teeth of the wheel.

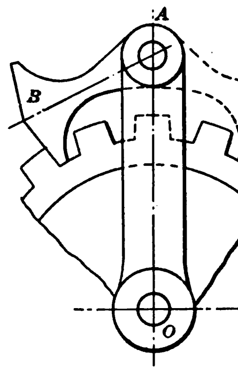


FIG. 156.

In Fig. 157, if the full movement of the pawl will advance the wheel, say eight teeth, then the cam *b*, which has a slightly larger diameter than the outside diameter of the wheel, is cut away on one side so that the pawl can act on the wheel tooth

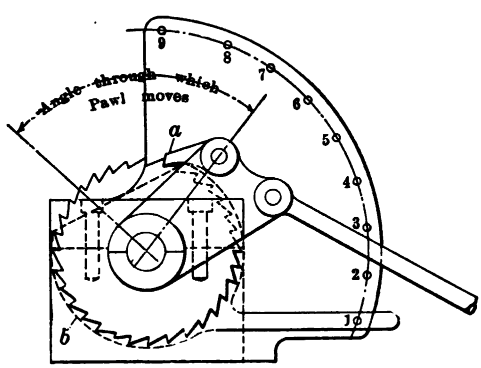


FIG. 157.

during the whole angle, and hole number 1 is located in the plate. Then the cam is revolved on its shaft so that the pawl will act on the teeth during the last seven-eights of its motion, and hole number 2 is located, and so on, locating the

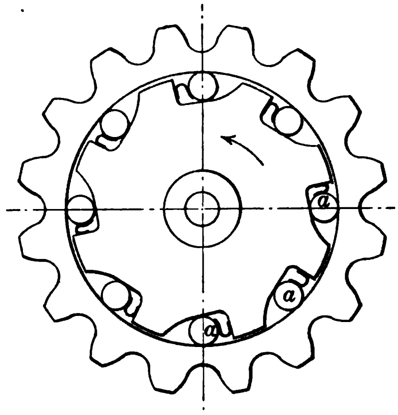


FIG. 158.

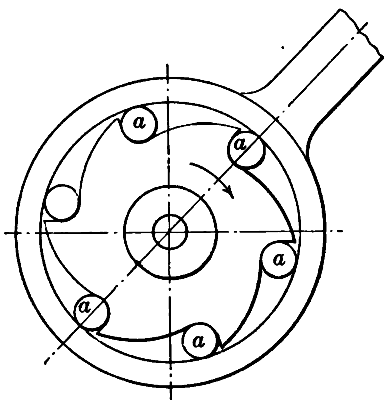


FIG. 159.

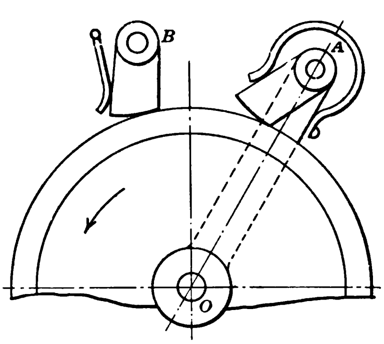


FIG. 160.

holes so that one tooth will be cut out each time, till in the ninth hole all of the teeth are masked. The width of pawl face must be equal to the face of ratchet wheel plus the cam face. A mechanism similar to this is used on some of the positive feed engine lubricators and on printing press inking mechanisms.

154. It is not always necessary that the connection between the wheel and pawl be of positive form as shown in the foregoing figures, and in some cases a friction drive may be substituted as in Figs. 158, 159 or 160. These are sometimes called *silent* ratchets.

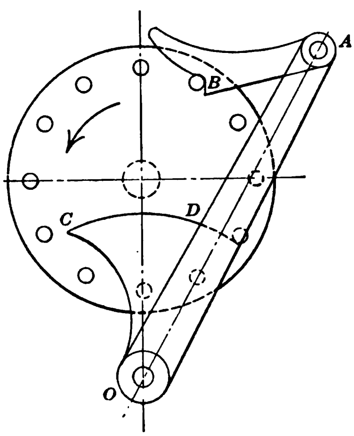


FIG. 161.

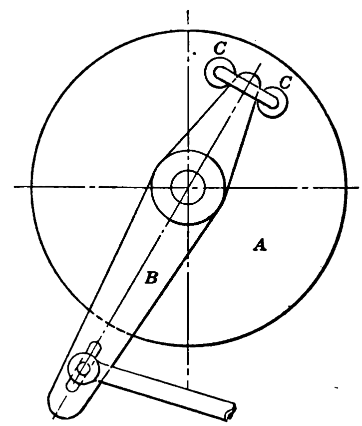


FIG. 162.

155. The ratchet mechanisms so far discussed together with many variations of them are suitable for feed mechanisms where the motions of the driver are not too rapid; in such cases the shock at the beginning of motion may be too great, and the inertia of the wheel may be such as to cause it to "over-travel." In such mechanisms as revolution counters where a definite motion of the follower must be secured each time, some means must be employed to prevent over-travel. A device for this purpose is shown in Fig. 161. The lever to which the pawl is attached has a projecting beak so formed that when the pawl first acts on a pin, the end C of this beak passes across the line of motion of the pins and limits their motion. The curve CD is an arc with O as a center.

The magnetic ratchet shown in Fig. 162 also accomplishes the same result. This consists of a metal disk A and an arm B that

swings about the center of A. On B is a magnet C, which grips the plate and by a commutator device is released for the return stroke of the lever.

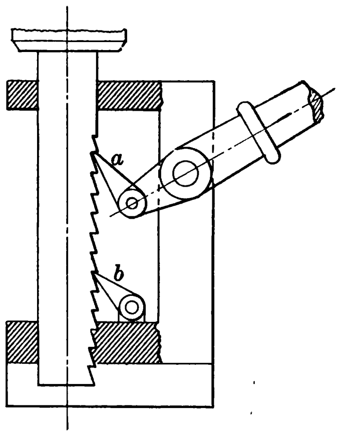


Fig. 163.

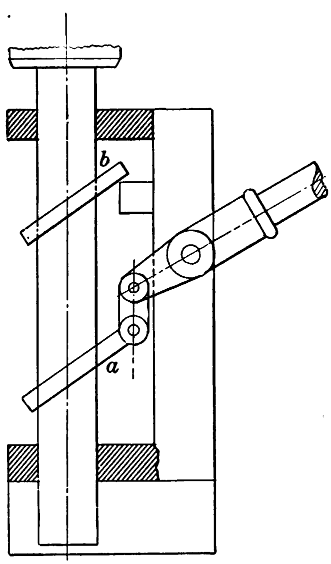


Fig. 164.

As soon as the magnet C releases A a second magnet on the back (not shown), grips it and prevents any backward movement of the disk. These ratchets are used on dividing engines.

156. In all of the ratchet mechanisms so far discussed, a

rotary motion was given to the wheel, but ratchet mechanisms can be used to impart a reciprocating motion. Two examples of this kind are shown in the lifting jacks of Figs. 163 and 164. In each case, the pawl *a* does the lifting, and *b* holds the load while a new grip is taken with *a*. Fig. 164 is of the silent ratchet type.

Clutches

157. In Fig. 165 is shown a method of connecting the ends of two shafts, *A* and *B* that are in alignment so that motion can be

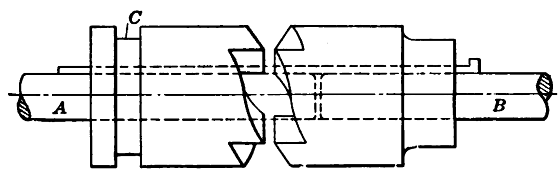


Fig. 165.

transmitted from *B* to *A* in one direction only. This is accomplished by having the part of the clutch on *B* rigidly keyed to it, and the part on *A* fastened by a feather key to prevent its turning, but allowing it to be moved along the shaft, to engage with the part on *B*. These jaws can be made the other hand from that shown so that *A* will be turned in the opposite direction.

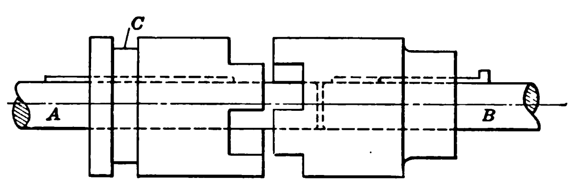


Fig. 166.

If it is desired to have *A* turn in either direction the jaws can be made square as shown in Fig. 166. These are positive clutches.

Another type of clutch is that in which the motion is imparted from one part to the other by means of friction between the surfaces.

One of the most common forms of this type is shown in Fig. 167 and is known as a cone clutch. Another form is shown in Fig. 168, in which the friction surfaces are alternate disks of

different materials and are brought into contact by means of the toggle joint arrangement *K.L.M.*

In the figure this clutch is shown on a single shaft. When the clutch is "thrown out" the part *A* which has a pulley fastened

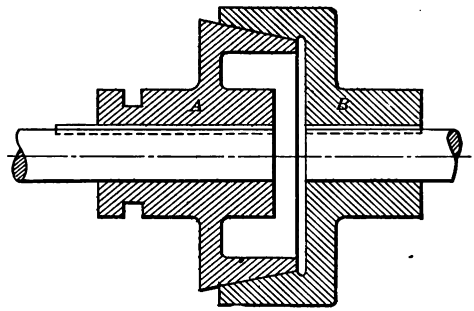


Fig. 167.

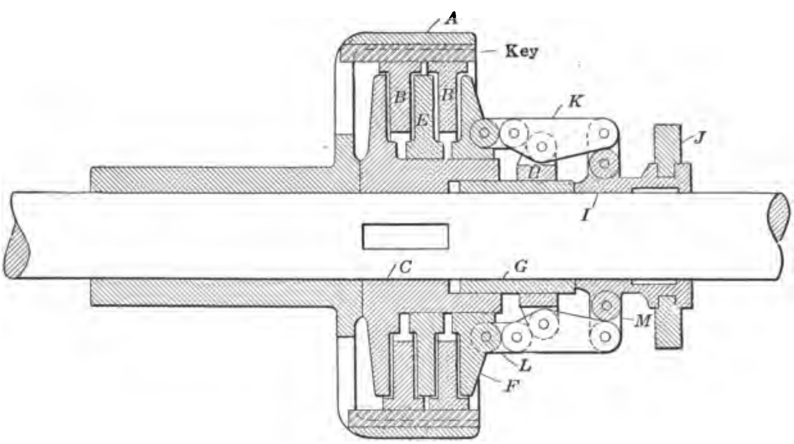


Fig. 168.

on its long hub, but not shown in the figure, does not revolve, but is made to revolve by the action of the toggle joint, bringing the friction surfaces into contact.

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